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Review paper

ANGULAR PARAMETERS OF UNDERMINED GROUND MOVEMENT PROCESS IN UNKNOWN AREAS

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Abstract: Angular parameters of ground movement process are determined in practice through measurements, as independent quantities. Theoretical researches have shown interdependence of angular parameters has to stand. Based on the known angular parameters of ground movement process for certain geometric and geological characteristics of the deposits can be determined by the parameters and any other terms of deposits. The methodology presented in this paper can be determine angular parameters of undermined ground movement process in unknown area which have not been any instrumental observations, and can also be applied for control of already determined angular parameters.

Key words: ground movement process, angular parameters, limit angles

1. INTRODUCTION

Angular parameters of ground movement process are determined in practice through measurements, as independent quantities. However, since all of them are related to the same process, it is obvious that they are related. Theoretical researches have shown that in case of varying of mining and geological properties in the coal deposit (seam thickness, inclination and depth, roof properties), interdependence of angular parameters has to stand. Also, with increase of mining depth, variation of limit angles values has certain regularity (Đorđević, 1989).

Determination of angular parameters for mines in unknown areas, without surveying data, is usually performed in analogy with deposits which had similar mining and geological properties, and well determined ground movement process parameters.

If we have well established angular parameters for specific mining and geological properties, we could use their geometric relations, regularity of variation with increase of seam depth, horizontal and vertical reduction to determine angular parameters for any mining depth, seam inclination, seam thickness and roof properties (Vušović and Svrkota, 2005a).

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This way, we can determine parameters of ground movement process for any deposit in unknown area, with satisfying accuracy.

2. INTERDEPENDENCE OF ANGULAR PARAMETERS α, β, γ AND θ

Papers from this field show angular parameters of undermined ground movement process as independent quantities (Figure 1). However, since these angles are related to the same process in the ground, it is obvious that they are interdependent. Theoretical researches have shown their relation through following equation:

$$\tan\theta = \frac{\sin\beta + K\sin\gamma}{\cos\beta - K\cos\gamma} \tag{1}$$

where:

$$K = \sqrt{\frac{H_1 \sin(\alpha+\beta)\sin\beta}{H_2 \sin(\gamma-\alpha)\sin\gamma}}$$

For these angles we can determine that they are coherent. Coherent angles enable determination of angle θ and, when limit angle θ doesn't match the required condition $\alpha < \gamma$.

We can see that $\theta > \beta$, so if there are no irregularities in the ground, or in coal seam, it always has to be $\beta < \theta < 90°$. If $\alpha = 0$, then $H_1 / H_2 = 1$ and $\beta = \gamma = \delta$, which means that $K = 1$.

Following relation:

$$\tan\alpha + \tan\beta \le \tan\gamma \tag{2}$$

can be used for measurement quality control and evaluation of limit angles β and γ.

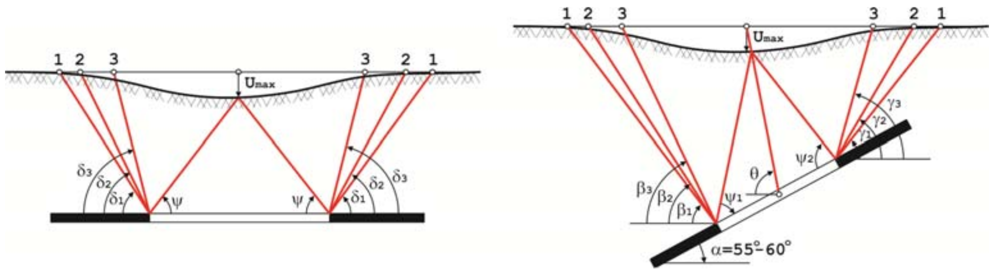

Figure 1 - Angular parameters of undermined ground movement process:
δ_1, β_1, γ_1 - limit angles; δ_2, β_2, γ_2 - safety angles; δ_3, β_3, γ_3 - fracture angles;
ψ, ψ_1, ψ_2 - full movement angles; θ - maximal subsidence angle; α - seam inclination

Angles β and γ, as interdependent quantities, can be determined through following equations:

$$\cot\beta = K_1\sqrt{\cot\alpha + \cot\theta + \frac{K_1^{\,2}}{4}} + \left(\cot\theta + \frac{K_1^{\,2}}{2}\right) \tag{3}$$

$$\cot\gamma = K_2\sqrt{\cot\alpha + \cot\theta + \frac{K_2^{\,2}}{4}} - \left(\cot\theta + \frac{K_2^{\,2}}{2}\right) \tag{4}$$

where:

$$K_1 = \frac{\tau_1}{\tau_2}\sqrt{h}\,\frac{\cot\theta + \cot\gamma}{\sqrt{\cot\alpha - \cot\gamma}} \quad \text{and} \quad K_2 = \frac{\tau_1}{\tau_2\sqrt{h}}\,\frac{\cot\beta - \cot\gamma}{\sqrt{\cot\alpha + \cot\beta}}$$

Finally:

$$q = \frac{\tau}{v_\alpha} \tag{5}$$

Considering the pair of coherent angles β and γ, after the calculation of angle θ using (1), we gain the same result for v_α, whether we calculate it using angles γ and θ:

$$v_\alpha = \sqrt{H_1}\,\frac{\cot\gamma + \cot\theta}{\sqrt{\cot\alpha - \cot\gamma}} \tag{6a}$$

or by using angles β and θ:

$$v_\alpha = \sqrt{H_2}\,\frac{\cot\beta - \cot\theta}{\sqrt{\cot\alpha + \cot\beta}} \tag{6b}$$

Even these relations include the depths H_1 and H_2, v_α is not dependable to them, because according to (5):

$$v_\alpha = \frac{\tau}{q} = const$$

In order to include the influence of relation U/U_o to limit angles, according to (5), we have to begin from general relation:

$$\frac{v_\alpha}{\tau} = \frac{v_{\alpha_o}}{\tau_o} \tag{7}$$

Calculation of limit angles β and γ comes to square equation, whose solution again has the form of (3) and (4), but generally it is:

$$K_1 = \frac{\tau v_\alpha}{\tau_o\sqrt{H_1}} \tag{8}$$

$$K_2 = \frac{\tau v_\alpha}{\tau_o\sqrt{H_2}} \tag{9}$$

3. TABLE OF PREDICTED VALUES OF LIMIT ANGLES

Limit angles are main parameters for prediction calculation of undermined ground parameters, and prediction accuracy depends on accuracy of limit angles. There are different criteria for their determination.

By first criterion, this is the angle between the line connecting end – point of underground stope with limit point at the surface (with values of subsidence

$U = 10$ mm, slope $N = 0{,}5$ mm/m and horizontal deformation $D = 0{,}5$ mm/m) and the horizontal, for critical area of extraction.

By second criterion, limit point at the surface is the point with 1-5% of maximal subsidence.

It is a general opinion that first criterion is less reliable, because for identical area of extraction, seam depth and roof properties, but different seam thickness, limit point with subsidence of $U = 10$ mm will not appear at the same place, which means that the values of limit angles will differ. Second criterion could also be inaccurate, because for identical percent of subsidence in limit point, but different extracted seam thicknesses, we get different values of subsidence. It is desirable that limit angles are determined in such manner that they could be comparable in any mine with any mining and geological properties.

Approximate values for limit angles and maximal subsidence angle, depending on seam inclination and ground properties are in Table 1 (Patarić and Stojanović, 1994). Each of these angles is gained separately, as independent quantity.

Table 1 - Prediction values of limit angles and maximal subsidence angle

Footwall is stable, theoretical thesis and equations for reduction are applicable										Footwall is unstable, theoretical thesis and equations are not applicable			
f	α=0		α=10°		α=20°	α=30°	α=40°	α=50°	α=55°	α=60°	α=70°	α=80°	α=90°
1.5	δ_1	54	β_1	47	42	37	33	30	29	28	27	26	25
			γ_1	57	60	64	67	71	73	73	57	40	25
			θ	85	81	78	76	74	74	73	75	80	90
	h_o	3.456	v_α	5.696	8.505	10.973	13.455	15.590	16.531	-	-	-	-
2.5	δ_1	60	β_1	53	47	41	37	33	32	29	28	27	26
			γ_1	63	66	69	71	74	75	76	59	41	26
			θ	85	81	77	75	73	73	74	73	79	90
	h_o	4.462	v_α	4.551	6.946	9.295	11.544	13.842	14.883	-	-	-	-
4.0	δ_1	66	β_1	59	52	46	40	36	34	32	29	28	27
			γ_1	70	72	74	76	77	78	79	61	42	27
			θ	84	80	76	73	72	71	72	74	77	90
	h_o	5.768	v_α	3.476	5.578	7.626	9.926	12.272	13.494	14.841	-	-	-
5.0	δ_1	70	β_1	62	54	47	42	37	35	33	32	31	29
			γ_1	73	75	77	79	80	81	80	64	43	28
			θ	84	79	75	72	70	69	70	72	77	90
	h_o	7.078	v_α	3.017	5.009	7.052	8.984	11.391	12.519	14.234	-	-	-
8.0	δ_1	75	β_1	66	58	50	44	39	36	34	32	31	29
			γ_1	79	82	84	85	86	85	84	67	44	29
			θ	83	78	73	69	67	66	67	70	76	90
	h_o	9.614	v_α	2.302	3.842	5.681	7.648	9.753	11.402	12.956	-	-	-

Approximate values of all angular parameters are included in a single table, where the values are more accurate and interdependent (Patarić and Stojanović, 1991). Beside limit angles, this table also provides auxiliary angles β, γ and δ, for mines with unknown ground movement process, for any seam thickness and any extraction depth.

Table was formed with all of the data reduced vertically – to the same depth and horizontally – to the same relation between limit subsidence and maximal subsidence, U_G/U_o (Patarić and Stojanović, 1994). Besides, values of angles β, γ and θ, are adjusted as interdependent quantities using equations (1), (3) and (4).

All of the equations can be used in case when footwall is stable and seam inclination α up to $\alpha \leq 55°$for weak rock and $\alpha \leq 60°$ for stronger rock.

Data in the table are given for following parameters: extraction depth $H_o = 300$ m, $U_G/U_o = 0.005$, maximal subsidence $U_o = 2000$ mm and subsidence at limit point $U_G = 10$ mm, with $\tau_o = 2{,}576$ (Table 2).

4. REDUCTION OF ANGULAR PARAMETERS

For different input values of extraction depth H and maximal subsidence U_o, for seam inclinations $\alpha \leq 55°$and $\alpha \leq 60°$, we can use horizontal and vertical reduction shown in equations (7), (8) and (9) to get the values of angular parameters (Đorđević, 2007).

Horizontal reduction means derivation of known limit angle value for specific point, in per mill of maximal subsidence, to another limit point, with random value of conditional subsidence.

Vertical reduction means derivation of limit angle from specific depth to another random depth.

4.1. Horizontal reduction

This reduction is significant in a sense of determination of limit angles for variable seam thicknesses, i.e. for variable maximal subsidence at the surface.

Limit subsidence in the table of prediction values (Table 1) is $U_G = 10$ mm. This value provides different limit angles for different maximal subsidence U_o. Limit angles are identical if the relation U_G/U_o stays unchanged.

With data provided in Table 1, it is possible to calculate limit angles for any value of U_o, using the table with values of function $\Phi_{(\tau)}$ (Table 2):

$$\Phi_{(\tau)} = \frac{2}{\sqrt{2\pi}} \int_o^{\tau} e^{-\frac{1}{2}t^2} dt \qquad (10)$$

which is familiar in mathematic statistics. Each value U_G/U_o matches a value of τ, calculated in following equation:

$$1 - 2\frac{U_G}{U_o} = \Phi_{(\tau)} \qquad (11)$$

4.1.1. Horizontal reduction of flat coal seams

In this case extraction depth is $H = 300$ m, which is the condition for values of δ_I, given in table 1.

Table 2 - Function $\Phi_{(\tau)} = \frac{2}{\sqrt{2\pi}} \int_o^{\tau} e^{-\frac{1}{2}t^2} dt$

τ	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0080	0.0160	0.0239	0.0319	0.399	0.478	0.0558	0.0638	0.0.717
0.1	0.0797	0.0876	0.0955	0.1034	0.1113	0.1192	0.1271	0.1350	0.1428	0.1561
0.2	0.1585	0.1663	0.1741	0.1819	0.1897	0.1974	0.2051	0.2128	0.2205	0.2282
0.3	0.2358	0.2434	0.2510	0.2586	0.2661	0.2737	0.2812	0.2886	0.2960	0.3035
0.4	0.3108	0.3182	0.3255	0.3328	0.3401	0.3473	0.3545	0.3616	0.3688	0.3759
0.5	0.3829	0.3900	0.3969	0.4039	0.4108	0.4177	0.4245	0.4313	0.4381	0.4448
0.6	0.4515	0.4581	0.4647	0.4713	0.4778	0.4843	0.4908	0.4971	0.5035	0.5098
0.7	0.5161	0.5223	0.5285	0.5346	0.5407	0.5468	0.5528	0.5587	0.5646	0.5705
0.8	0.5763	0.5821	0.5878	0.5935	0.5991	0.6047	0.6102	0.6157	0.6211	0.6265
0.9	0.6319	0.6372	0.6424	0.6476	0.6528	0.6579	0.6629	0.6680	0.6729	0.6778
1.0	0.6827	0.6875	0.6923	0.6970	0.7017	0.7063	0.7109	0.7154	0.7199	0.7243
1.1	0.7287	0.7330	0.7373	0.7415	0.7457	0.7499	0.7540	0.7580	0.7620	0.7660
1.2	0.7699	0.7737	0.7775	0.7850	0.7887	0.7905	0.7923	0.7959	0.7994	0.8030
1.3	0.8064	0.8098	0.8132	0.8165	0.8198	0.8230	0.8262	0.8293	0.8324	0.8355
1.4	0.8385	0.8415	0.8444	0.8473	0.8501	0.8529	0.8557	0.8584	0.8611	0.8638
1.5	0.8664	0.8690	0.8715	0.8740	0.8764	0.8789	0.8812	0.8836	0.8859	0.8882
1.6	0.8904	0.8926	0.8948	0.8969	0.8990	0.9011	0.9031	0.9051	0.9070	0.9090
1.7	0.9109	0.9127	0.9146	0.9164	0.9181	0.9199	0.9216	0.9233	0.9249	0.9266
1.8	0.9281	0.9297	0.9312	0.9328	0.9342	0.9357	0.9371	0.9385	0.9399	0.9412
1.9	0.9426	0.9439	0.9451	0.9464	0.9476	0.9488	0.9500	0.9512	0.9523	0.9534
2.0	0.9545	0.9556	0.9566	0.9576	0.9586	0.9596	0.9606	0.9616	0.9625	0.9634
2.1	0.9643	0.9651	0.9660	0.9668	0.9676	0.9684	0.9692	0.9700	0.9707	0.9715
2.2	0.9722	0.9729	0.9736	0.9742	0.9749	0.9756	0.9762	0.9768	0.9774	0.9780
2.3	0.9786	0.9791	0.9797	0.9802	0.9807	0.9812	0.9817	0.9822	0.9827	0.9832
2.4	0.9836	0.9840	0.9845	0.9849	0.9853	0.9857	0.9861	0.9865	0.9869	0.9872
2.5	0.9876	0.9879	0.9883	0.9886	0.9889	0.9892	0.9895	0.9898	0.9901	0.9904
2.6	0.9907	0.9910	0.9912	0.9915	0.9917	0.9920	0.9922	0.9924	0.9926	0.9929
2.7	0.9931	0.9933	0.9935	0.9937	0.9939	0.9940	0.9942	0.9943	0.9946	0.9947
2.8	0.9949	0.9950	0.9952	0.9953	0.9955	0.9956	0.9958	0.9959	0.9960	0.9962
2.9	0.9963	0.9964	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972
3.0	0.9973	0.9974	0.9975	0.9976	0.9976	0.9977	0.9978	0.9979	0.9979	0.9980
3.1	0.9981	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986
3.2	0.9986	0.9987	0.9987	0.9988	0.9988	0.9988	0.9989	0.9989	0.9990	0.9990
3.3	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.4	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.9995
3.5	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997
3.6	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998	0.9998	0.9998
3.7	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Horizontal reduction of flat seams is provided by equation (12):

$$\frac{\tau \cdot tg\delta}{\sqrt{H}} = \frac{\tau_o \cdot tg\delta_o}{\sqrt{H_o}} = const \qquad (12)$$

In horizontal reduction the depth is not changing, so we have:

$$\tau \cdot \tan\delta = h_o = const \tag{13}$$

In order to make the calculation easier, table also provides values of $h_o = \tau_o \cdot \tan\delta_o$ for 5‰ limit in a column for $\alpha = 0$ (Vušović and Svrkota, 2005b).

Example No. 1.

The task is to perform a horizontal reduction for $f = 4$ (Protodjakonov's rock strength coefficient) and $U_o = 2500$ mm.

Solution:

Table 1 is based on relation $U_G/U_o = 0.005$, with $\tau_o = 2.576$, while for $f = 4$, we get $\delta_o = 65°$, so it is:

$$h_o = 2.576 \cdot \tan 65^o = 5.52$$

For limit subsidence of $U_G = 10$ mm and maximal subsidence of $U_o = 2500$ mm we have:

$$\Phi(\tau) = 1 - 2 \cdot \frac{10}{2500} = 0.992$$

With corresponding $\tau = 2.65$. Finally, we have:

$$\tan\delta_1 = \frac{h_o}{\tau} = \frac{5.52}{2.65} \quad \Rightarrow \quad \delta_1 = 64^o 20'$$

4.1.2. Horizontal reduction of inclined coal seams

In case of inclined seam, subsidence curve is asymmetric, with maximal subsidence point moved in direction of seam descent. Position of maximal subsidence point is determined by angle of maximal subsidence θ. Values of limit angles in direction of seam descent β_l and ascent γ_l are different. Table 1 provides the values of these angles for variable seam inclination, fixed limit subsidence $U_G = 10$ mm and reduced to the fixed depth of 300 m.

Angles β_l, γ_l and θ are not independent. If two of them are known, the third could be derived. For example, if we know limit angles β_l and γ_l, apply equation (1). This is important, because in calculation of coefficients v_α, necessary for reduction, the result must be the same, regardless on equation in use (6a or 6b).

For input table data values of v_α are given in order to avoid the errors as the consequence of approximation of limit angles values, and to make calculation easier. In interpolation of limit angles values which are not given in the table, the procedure is to interpolate the values of β_l and γ_l, and then use them to calculate θ, thus providing calculation of v_α.

When the angles α and θ, are known, each value of $v/\sqrt{H}$ has corresponding limit angles β_l and γ_l. For instance, if we have:

$$\frac{v}{\sqrt{H}} = K$$

based on (6b) it is:

$$\frac{v}{\sqrt{H}} = \frac{\cot\beta - \cot\theta}{\sqrt{\cot\alpha + \cot\beta}} = K$$

We can derive $\cot\beta$ in square equation:

$$\left(\cot\beta - \cot\theta\right)^2 = K^2 \cdot \left(\cot\alpha + \cot\beta\right)$$

whose solution is angle β (3). On the same way, from (6a) we calculate angle α (4).

For inclined seams, we have following relation:

$$\frac{v}{\tau} = \frac{v_\alpha}{\tau_o} = const \tag{14}$$

Since there is no change of depth in horizontal reduction, relation (14) can be expressed as:

$$\frac{v}{\tau \cdot \sqrt{H}} = \frac{v_\alpha}{\tau_o \cdot \sqrt{H_o}} = h_\alpha$$

In order to make calculation easier, table 1 gives values for h_α, along with v_α, so general equation for horizontal reduction of inclined seam is:

$$\frac{v}{\sqrt{H}} = \tau \cdot h_\alpha \tag{15}$$

Limit angles β_l and γ_l are calculated by formulas (3) and (4), where coefficient K is calculated by:

$$K_h = \tau \cdot h_\alpha \tag{16}$$

Example No. 2.

The task is to determine limit angle γ_l at the upper part of subsidence curve, if seam inclination is $\alpha = 30°$, depth $H_o = 300$ m, $U_o = 2500$ mm, $f = 4$.

Solution:

Like in Example No.1, we have $\tau_o = 2.576$, while in Table 1 for $f = 4$ and $\alpha = 30°$, we find $\theta = 76°$ and $v_\alpha = 7.626$. After that, we calculate:

$$h_\alpha = \frac{v_\alpha}{\tau_o \cdot \sqrt{H_o}} = \frac{7.626}{2.576 \cdot \sqrt{300}} = 0.1709$$

$$K_2 = \tau \cdot h_\alpha = 2.65 \cdot 0.1709 = 0.453$$

so it is:

$$\cot\gamma = 0.453 \cdot \sqrt{\cot 30^o + \cot 76^o + \frac{0.453^2}{4}} - \left(\cot 76^o + \frac{0.453^2}{2}\right) = 0.2939$$

$$\gamma = 73.6^o$$

4.1.3. Vertical reduction of flat seams

Values of limit angles are increasing with the depth, so the task is to reduce their values from $H_o = 300$ m to any random depth. This is done by vertical reduction.

In vertical reduction, parameter τ remains constant, so from (12) we have a relation independent to variation of depth:

$$\frac{\tan\delta_1}{\sqrt{H}} = v_o$$

This relation is easy for application due to its simplicity.

Example No. 3.

The task is determination of limit angle δ_l for the seam laying at $H = 500$ m, with maximal subsidence $U_o = 2500$ mm and $f = 4$.

Solution:

Relation U_G/U_o is 5‰, same as input value in Table 1, so it is:

$$v_o = \frac{\tan 65^o}{\sqrt{300}} = 0.1238$$

$$\tan\delta_1 = v_o \cdot \sqrt{H} = 0.1238 \cdot \sqrt{500} = 2.768 \quad \Rightarrow \quad \delta_1 = 70^o$$

4.1.4. Vertical reduction of inclined seams

In vertical reduction we have $\tau = \tau_o$, so based on (14) it is:

$$v = v_\alpha = const$$

Or, to derive it to square equation:

$$\frac{v}{\sqrt{H}} = \frac{v_\alpha}{\sqrt{H}}$$

Limit angles β_l and γ_l are calculated by (3) and (4), whilst for coefficient K it is:

- In (3): $$K_{\beta v} = \frac{v_\alpha}{\sqrt{H_2}} \tag{17}$$
- In (4): $$K_{\gamma v} = \frac{v_\alpha}{\sqrt{H_1}} \tag{18}$$

By the way, (6a) and (6b) provide the possibility to calculate one limit angle when we know the value of the second limit angle and maximal subsidence angle θ.

Example No. 4.

The task is to determine limit angles β_l and γ_l for seam laying at $\alpha = 30°$, $U_G/U_o = 0.005$, $H_l = 250$ m, $H_2 = 350$ m and $f = 2.5$.

Solution:

From Table 1 we find $\theta = 77°$ and $v_\alpha = 9.295$. With:

$$K_{\gamma v}=\frac{9.295}{\sqrt{250}}=0.588$$

according to (4), it is:

$$\cot\gamma_1=0.588\cdot\sqrt{\cot 30^o+\cot 77^o+\frac{0.588^2}{4}}-\left(\cot 77^o+\frac{0.588^2}{2}\right)=0.4383$$

$$\gamma_1=66.3^o$$

With:

$$K_{\beta v}=\frac{9.295}{\sqrt{350}}=0.497$$

according to (3), it is:

$$\cot\beta_1=0.497\cdot\sqrt{\cot 30^o+\cot 77^o+\frac{0.497^2}{4}}+\cot 77^o+\frac{0.497^2}{2}=1.062$$

$$\beta_1=43.3^o$$

4.1.5. Direct reduction of flat seams

In cases when relation U_G/U_o and the depth don't match table values, both horizontal and vertical reduction are needed.

Limit angles are determined through direct reduction.

For horizontal reduction $H=\text{const}$, and τ is variable. For vertical reduction, $\tau=\text{const}$, while H varies.

According to (12), for flat seam it is:

$$\tan\delta=\frac{\sqrt{H}}{\sqrt{H_o}}\cdot\frac{h_o}{\tau} \qquad (19)$$

This equation, in horizontal reduction, when $H=H_o$, derives to equation (13).

By applying equation (19), we can directly determine limit angle for the depth $H\neq H_o$ and specified relation U_G/U_o.

Example No. 5.

The task is to determine limit angle δ_l, with $H=500$ m, $U_o=2500$ mm and $f=5$.

Solution:

Similar as in Example No.1, we calculate $\tau=2.65$, while for $f=5$, and using h_o from Table 1, according to (13), we have:

$$\tan\delta_1=\frac{\sqrt{500}}{\sqrt{300}}\cdot\frac{7.078}{2.65}=3.448 \quad\Rightarrow\quad \delta_1=73.8^o$$

4.1.6. Direct reduction of inclined seams

From (14), we have:

$$\frac{v}{\sqrt{H}} = \frac{\tau}{\tau_o} \cdot \frac{v_\alpha}{\sqrt{H_o}}$$

which means that (3) and (4) are usable again, while coefficient K is in (3):

$$K_{\beta d} = \frac{\tau}{\tau_o} \cdot \frac{v_o}{\sqrt{H_2}} \qquad (20)$$

and in (4):

$$K_{\gamma d} = \frac{\tau}{\tau_o} \cdot \frac{v_o}{\sqrt{H_1}} \qquad (21)$$

Example No. 6.

The task is determination of limit angle in the lower part of subsidence curve, with $\alpha = 30°$, $H_2 = 400$ m, $U_o = 1000$ mm and $f = 2.5$.

Solution:

Similar to Example No.1, it is:

$$\Phi(\tau) = 1 - 2 \cdot \frac{10}{1000} 0.9800$$

consequently:

$$\tau = 2.326$$

From Table 1 we find $\theta = 77°$ and $v_\alpha = 9.295$. This is input data for (20):

$$K_{\beta d} = \frac{2.326}{2.576} \cdot \frac{9.295}{\sqrt{400}} = 0.420$$

followed by (3):

$$\cot \beta_1 = 0.420 \cdot \sqrt{\cot 30^o + \cot 77^o + \frac{0.420^2}{4}} + \cot 77^o + \frac{0.420^2}{2} = 0.9142$$

$$\beta_1 = 47.6^o$$

5. CONCLUSION

Methodology of determination of angular parameters presented in this paper can be successfully applied in mines with unknown ground movement process, if mining and geological properties are regular (seam thickness, seam inclination, tectonics, etc.). Besides, this method can also be applied for control of already determined angular parameters, or determination of regularity of mining and geological properties in the deposit. Results provided by this method have the accuracy needed for prediction of ground movement process and protection of structural objects at the surface from underground mining.

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