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QUANTIFICATION OF UNCERTAINTY IN MINING PROJECT RELATED TO METAL PRICE USING MEAN REVERSION PROCESS AND INTERVAL TYPE-2 FUZZY SETS THEORY

KVANTIFIKACIJA NEODREĐENOSTI CENE METALA U RUDARSKOM PROJEKTU PRIMENOM PROCESA POVRATKA NA SREDNJU VREDNOST I TEORIJE INTERVALNO RASPLINUTIH SKUPOVA DRUGOG TIPA

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Abstract: Real world mine project value estimation is ill defined, i.e., its parameters are not precisely known. Estimating future mineral prices - particularly prices far enough into the future to be of use in mine investment analysis - is an exercise for which a high error of estimation invariably exists. The characteristically long preproduction periods of mining projects mean that success of these capital-intensive ventures will be determined by mineral prices five to ten years in the future. The market risks related to metal price are modelled with a special stochastic process, a Mean reversion process. Validity of the parameters of the Mean reversion process directly depends on source of information. Parameters of the Mean reversion process are defined as follows: the speed of mean reversion k is fixed, the long-run equilibrium metal price $\overline{P}$ is fixed, volatility σ is defined by lower and upper bound and its variation in this interval is uniform and constant over time. To decrease uncertainty we firstly make simulation of future states of metal price and after that simulated values are converted to interval type-2 fuzzy triangular numbers.

Key words: mining project, uncertainty, metal price, mean reversion process simulation, interval type-2 fuzzy sets

Apstrakt: Određivanje vrednosti rudarskog projekta, u realnom svetu, je „nezdravo“ definisana, tj. parametri procene nisu precizno znani. Procenjivanje budućih cena metala-posebno cena u dalekoj budućnosti koje se koriste u rudarskim investicionim analizama - predstavlja vežbanje u kojem je nepromenljivo zastupljena velika greška procenjivanja. Svojstveno dugi periodi pre početka realizacije rudarskih projekata znače da će uspeh ovih skupih poduhvata biti određen cenama mineralnih sirovina u budućih pet do deset godina. Tržišni rizici vezani za cenu metala modeliraju se posebnim stohastičkim procesom, procesom povratka na srednju vrednost. Validnost parametara procesa povratka na srednju vrednost direktno zavisi od izvora informacija. Parametri procesa povratka na srednju vrednost definisani su na sledeći način: brzina

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povratka na srednju vrednost k je konstantna, dugoročna srednja cena metala $\overline{P}$ je konstantna, varijabilnost σ definisana je donjom i gornjom granicom, a njeno variranje unutar ovog intervala ima uniformnu raspodelu i konstantna je tokom vremena. Kako bi smanjili neodređenost, prvo izvodimo simulacije budućih stanja cene metala, a nakon toga, simulirane vrednosti konvertujemo u intervalno rasplinute trouglaste brojeve drugog tipa.

Ključne reči: rudarski projekat, neodređenost, cena metala, proces povratka na srednju vrednost, intervalno rasplinuti skupovi drugog tipa

1. INTRODUCTION

A typical mining operation presents risks and challenges in all its aspects, including evaluation, finance and construction. Decision pertaining to a firm's proposed capital investments can have vital short and long-term consequences on the company's ability to compete, and even survive. The evaluation of mining projects in today's environment is much complex than it was just a few years ago. There are typically myriads of variables which are directly or indirectly associated with the mine project evaluation process. As such, underground mine project valuation has become truly interdisciplinary in nature. Estimating mineral project revenue is indeed a difficult and risky activity. Annual mine revenue is calculated by multiplying the number of units produced and sold during the year by the sales price per unit.

Fluctuations in commodity prices are of interest because they affect the decisions taken by producers and consumers, because they play a crucial role in commodity-related investments, project appraisals and strategic planning, and because of the extent to which they reflect and influence general economic activity. In general, structural models are very useful because they provide valuable insights into the determinants of commodity price movements (Bernard et al. 2005).

Many authors conclude the existence of mean reversion for commodity prices. In examining stochastic models for commodity prices see some of them, for example, Schwartz and Smith (2000), Khalaf et al. (2003). Margaret Slade (2001) created a model of valuing managerial flexibility in mining investments, where price of natural-resource commodities is the sum of two stochastic components and both are potentially mean reverting. Michael Samis (2001) involved macro-economic uncertainty into flexible discrete mine production model through an uncertain mineral price that follows the Ornstein-Uhlenbeck process.

2. MODEL OF QUANTIFICATION

Estimating future mineral prices - particularly prices far enough into the future to be of use in mine investment analysis - is an exercise for which a high error of estimation invariably exists. The characteristically long preproduction periods of mining projects mean that success of these capital-intensive ventures will be determined by mineral prices five to ten years in the future.

The market risks related to commodity price are modelled with a special stochastic process, a Mean reversion process. The Mean reversion process has economic logic, for example, although the commodity prices have sensible short-term oscillations, they tend to revert back to a "normal" long-term equilibrium level. The mean reversion evidence is reported in many studies. The past values of the changes in this risk factor help predict the future. We will use a model where the metal spot price is assumed to follow the stochastic process (Schwartz, 1997):

$$dP = k\left(\ln \overline{P} - \ln P\right)Pdt + \sigma PdW \tag{1}$$

Let $x = \ln P$, applying Ito's Lemma allows the characterization of the log price by an Ornstein-Uhlenbeck stochastic mean reverting process:

$$dx = k\left(\overline{x} - x\right)dt + \sigma dW \tag{2}$$

with

$$\overline{x} = \ln\left(\overline{P}\right) - \frac{\sigma^2}{2k} \tag{3}$$

where are:

$\overline{P}$ - the long-run equilibrium metal price;

k - measures the speed of mean reversion to the long run mean log price $\overline{P}$;

dW - an increment to a standard Brownian motion;

σ - refers to the price volatility rate.

The correct discrete-time format for the continuous-time process of mean reversion is the stationary first order autoregressive process (Dixit and Pindyck, 1994), so the sample path simulation equation for x_t is performed by using exact discrete-time expression:

$$x_t = x_{t-1}e^{-k\Delta t} + x\left(1 - e^{-k\Delta t}\right) + N\left(0,1\right)\sigma\sqrt{\left(1 - e^{-2k\Delta t}\right)\Big/k} \tag{4}$$

where are:

Δt - the fixed time interval from time t to $t+1$;

$N(0,1)$ - the normally distributed random variable.

By substituting equation (4) to $P = e^x$, we have exact discrete-time equation for P_t, given by:

$$P_t = \exp\left\{\ln\left(P_{t-1}\right)e^{-k\Delta t} + \left[\ln\left(\overline{P}\right) - \frac{\sigma^2}{2k}\right]\left(1 - e^{-k\Delta t}\right) + N\left(0,1\right)\sigma\sqrt{\left(1 - e^{-2k\Delta t}\right)\Big/k}\right\} \tag{5}$$

In order to estimate the parameters of the Mean reversion process, we run the following regression.

$$dx_{t+1} = \beta_o + \beta_1 x_t + \varepsilon \tag{6}$$

where $\beta_o = k\overline{x}dt$ and $\beta_1 = -kdt$. Hence, if we regress observation dx against x, we can obtain estimates of β_o and β_1. And volatility σ is the standard deviation from the regression. By this way, obtained values of parameters are defined as precise values.

Let $P = \left\{P_t, t = 0,...,T\right\}$ denote a price scenario with spot prices P_t, where P_t is determined by equation (5). Figure 1 presents a sample paths ($s = 1, 2, ..., S$) of the commodity price (for example, zinc price) simulated using the above equation S times.

Figure 2 presents Probability density function (*pdf*) and Cumulative distribution function (*cdf*) of zinc price for $t = 1$ and $s = 1, 2, ..., S$, where S is the number of simulated price scenarious.

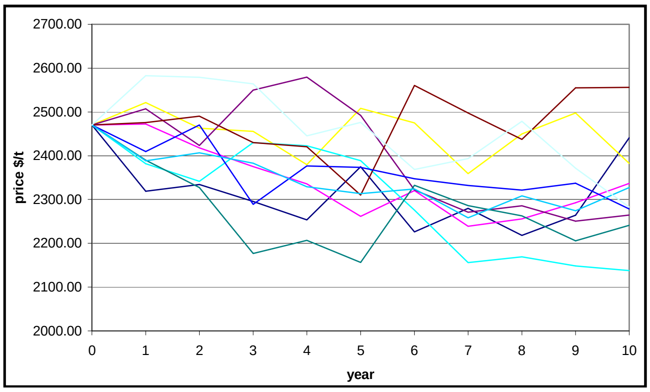

Figure 1 - Simulated zinc price paths on a yearly time resolution

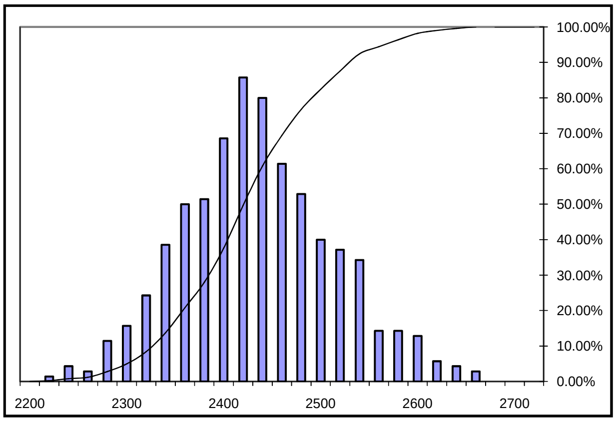

Figure 2 - Probability density function and Cumulative distribution function of zinc price for $t = 1$

Obviously, we obtain the distribution of metal price for every year of project time, i.e., $pdf_1^s, pdf_2^s, \ldots, pdf_T^s$, where $s = 1, 2, ..., S$.

In uncertainty analysis, variables usually are not single-valued but can assume a whole range of different values. The set collecting all possible values is denoted by Ω (the sure event). Hence, reliability analysis requires the confidence on the occurrence of subsets $A \subseteq \Omega$ be defined. A confidence measure is number $0 \le g(A) \le 1$, which represents the confidence one has on the occurrence of A. For more details see (Savoia, 2002; Ferrari and Savoia, 1998). Fuzzy numbers can be also used to express uncertainties related to input data. A fuzzy number $\tilde{M}$ is a convex normalized fuzzy set $\tilde{M}$ of the real line R such that (Bellman and Zadeh, 1977): it exists such that one $x_o \in R$ with $\mu_{\tilde{M}}(x_o)=1$ (x_o is called mean value of $\tilde{M}$); $\mu_{\tilde{M}}(x)$ is piecewise continuous.

There are many possibilities to use different fuzzy numbers according to the situation. Triangular fuzzy numbers (TFN) are very convenient to work with because of their computational simplicity and they are useful in promoting representation and information processing in fuzzy environment. In this paper, we use TFNs. Triangular fuzzy numbers can be defined as a triplet (*a*, *b*, *c*). The parameters *a*, *b* and *c* respectively, indicate the smallest possible value, the most promising value and the largest possible value that describe a fuzzy event. A fuzzy triangular number $\tilde{M}$ is shown in Figure 3.

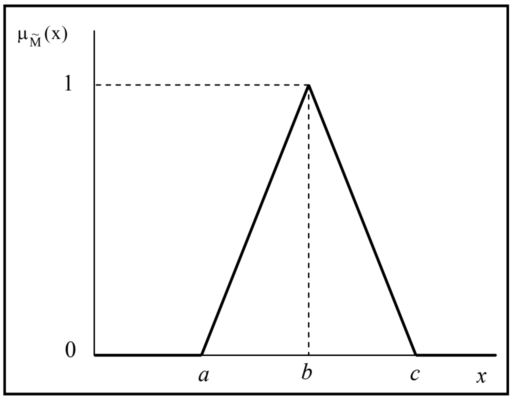

Figure 3 - Triangular fuzzy number

The membership function is defined as (Kaufmann and Gupta, 1985):

$$\mu_{\tilde{M}}(x)=\begin{cases}0, & x<a\\ \dfrac{x-a}{b-a}, & a\le x\le b\\ \dfrac{c-x}{c-b}, & b\le x\le c\\ 0, & x>c\end{cases} \tag{7}$$

Following way of transformation is used to convert uncertainty expressed by probability density function into uncertainty expressed by fuzzy number. Suppose we have unimodal continuous probability density function (*pdf*) *p* with bounded support [*a*, *c*], such that *p* is increasing on [*a*, *b*] and decreasing on [*b*, *c*] where *b* is the modal value of *p*. Define a function f:[*a*, *b*] → [*b*, *c*] by

$$f(x)=\max\{y\,|\,p(y)\ge p(x)\} \tag{8}$$

Then, the possibility distribution μ can be defined by

$$\mu(x) = \mu(f(x)) = \int_{-\infty}^{x} p(y)\,dy + \int_{f(x)}^{+\infty} p(y)\,dy \qquad (9)$$

If we denote the cumulative distribution function by

$$P(x) := P(X \le x) = \int_{-\infty}^{x} p(y)\,dy \qquad (10)$$

then by (9)

$$\mu(x) = P(x) + 1 - P(f(x)) \qquad (11)$$

The above idea can be illustrated as in Figure 4. For a random variable *X* with probability density function shown as in Figure 4a, for any $x \in [a,b]$, $s_1 = P(x)$ and $s_2 = 1 - P(f(x))$, then the possibility function $\mu(x) = \mu(f(x)) = \alpha = s_1 + s_2$ as shown in Figure 4b. As you can see, the support of the membership function and the *pdf* are the same, and the point with higher probability (likelihood) has the higher possibility. For more details see (Swishchuk et al. 2008).

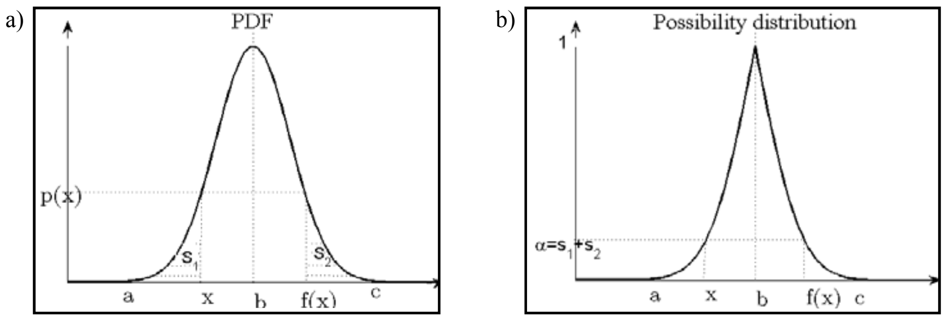

Figure 4 - Transformation from probability (a); to possibility (b)

Further, the uncertainty of input data can then be approximately modelled by a fuzzy triangular number. Without loss of generality let us consider one uncertain parameter characterized by the set of measured values and values show Gaussian distribution, i.e. Normal distribution. The uncertainty in the parameter is modelled by a fuzzy triangular number with the membership function which has the support of $\eta - 2\sigma < X < \eta + 2\sigma$ set up for around 95% confidence interval of normal distribution function. If we take into consideration the triangular fuzzy number is defined as a triplet (*a*, *b*, *c*) then *a* and *c* is lower bound and upper bound that obtained from lower and upper bound of 5% of the normal distribution and the most promising value *b* is equal to mean value of the distribution (Do et al. 2005). Figure 5 shows the illustration of transformation.

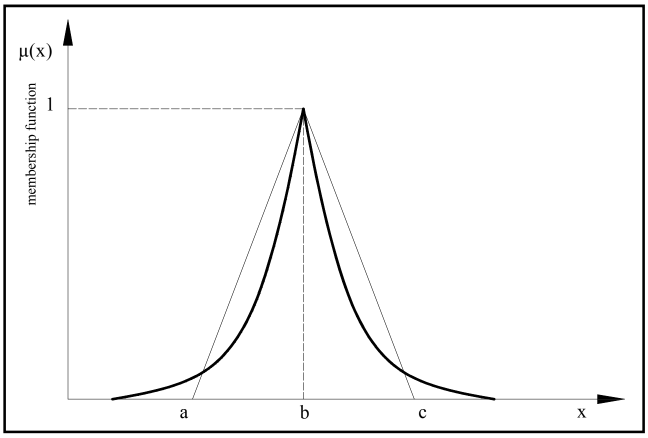

Figure 5 - The transformation from possibility to fuzzy triangular number

According to above way of transformation, metal price for every year of the project time is obtained as fuzzy triangular number, $\tilde{P}_1 = (a_1, b_1, c_1)$, $\tilde{P}_2 = (a_2, b_2, c_2)$, …, $\tilde{P}_T = (a_T, b_T, c_T)$.

Validity of the parameters of the Mean reversion process directly depends on source of information. Selection of real source plays a key role in the process of estimation of parameters. To make this process easier we propose the following approach.

In the world of mining business there are many sources of data related to metal prices. Without loss of generality we propose regress observation of three sources. Let M_1, M_2 and M_3 be a sets of metal prices related to first, second and third source respectively. Since regress observation has been done we obtained the following parameters: $\beta_{o(M_1)}, \beta_{o(M_2)}, \beta_{o(M_3)}, \beta_{1(M_1)}, \beta_{1(M_2)}, \beta_{1(M_3)}$. Further, suppose there are following relations between obtained parameters: $\beta_{o(M_1)} \le \beta_{o(M_2)} \le \beta_{o(M_3)}$, $\beta_{1(M_1)} \le \beta_{1(M_2)} \le \beta_{1(M_3)}$.

According to above facts, parameters of the Mean reversion process can be expressed as interval numbers. An interval number is an order pair of real numbers, $[a_1, a_2]$ with $a_1 \le a_2$. It is also a set of real numbers defined by:

$$[a_1, a_2] = \{x | a_1 \le x \le a_2\} \tag{12}$$

Obviously, the notion of interval numbers provides a tool for representing a real number by specifying its lower (L) and upper (U) endpoints (Yao, 1993). We can perform arithmetic with interval numbers through the arithmetic operations on their numbers. We can derive following formulas:

$$[a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2] \tag{13}$$

$$[a_1, a_2] - [b_1, b_2] = [a_1 - b_2, a_2 - b_1] \tag{14}$$

$$[a_1, a_2] \cdot [b_1, b_2] = [\min(a_1b_1, a_1b_2, a_2b_1, a_2b_2), \max(a_1b_1, a_1b_2, a_2b_1, a_2b_2)] \tag{15}$$

$$[a_1, a_2]/[b_1, b_2] = [a_1, a_2] \cdot [1/b_2, 1/b_1] \quad \text{for} \quad 0 \in [b_1, b_2] \tag{16}$$

Parameters of the Mean reversion process can be defined as following interval numbers:

$$\beta_o = [\beta_{oL}, \beta_{oU}];\ \beta_1 = [\beta_{1L}, \beta_{1U}] \tag{17}$$

where: $\beta_{oL} = \min\left(\beta_{o(M_1)}, \beta_{o(M_2)}, \beta_{o(M_3)}\right)$, $\beta_{oU} = \max\left(\beta_{o(M_1)}, \beta_{o(M_2)}, \beta_{o(M_3)}\right)$,

$\beta_{1L} = \min\left(\beta_{1(M_1)}, \beta_{1(M_2)}, \beta_{1(M_3)}\right)$ and $\beta_{1U} = \max\left(\beta_{1(M_1)}, \beta_{1(M_2)}, \beta_{1(M_3)}\right)$

However, we are often faced with incompletely knowledge about set of historical data and because of it, determination of parameters as precise values can be very hard task. Avellaneda et al. (1995) developed a model for pricing and hedging derivative securities and option portfolios in an environment where the volatility is not known precisely, but is assume instead to lie between two extreme values σ_{min} and σ_{max} [15]. To predict the future states of P_t we applied the same approach with following assumptions: the speed of mean reversion k is fixed, $k = -(\beta_{1L} + \beta_{1U})/2$; the long-run equilibrium metal price $\overline{P}$ is fixed, $\overline{P} = (\beta_{oL} + \beta_{oU})/2k$; volatility $\sigma \in [\sigma_L, \sigma_U]$, where σ_L and σ_U represent lower and upper bounds on the volatility, i.e., *min* and *max* value of volatility; σ_L and σ_U are constant over time; volatility vary anywhere in this interval and variation is uniform. According to above assumptions we consider the use of Mean reversion process (MRP) with a fixed speed of mean reversion k and long-run equilibrium metal price $\overline{P}$, but uncertain volatility σ in the following form:

$$P_t = \exp\left\{\ln(P_{t-1})e^{-k\Delta t} + \left[\ln(\overline{P}) - \frac{\sigma^2}{2k}\right]\left(1 - e^{-k\Delta t}\right) + N(0,1)\sigma\sqrt{\left(1 - e^{-2k\Delta t}\right)/2k}\right\} \tag{18}$$
$$\sigma \in [\sigma_L, \sigma_U]$$

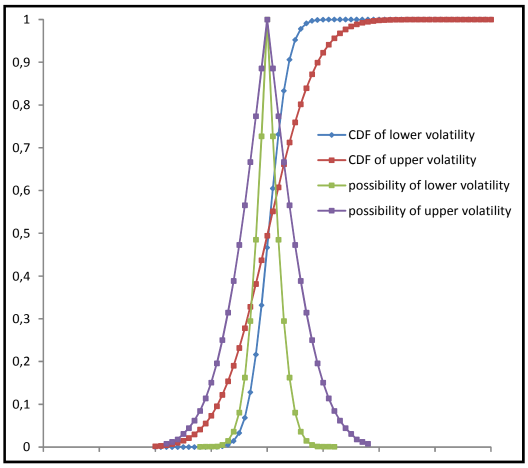

Figure 6 - Cumulative distribution functions of the metal price for $P_{t,L}$ and $P_{t,U}$; Possibility functions of the metal price for $P_{t,L}$ and $P_{t,U}$

On the base of *pdf* of $P_{t,L} \sim MRP\left(k,\overline{P},\sigma_L\right)$ and *pdf* of $P_{t,U} \sim MRP\left(k,\overline{P},\sigma_U\right)$ obtained by simulation, it is created two cumulative distribution functions. According to way of transformation (from probability to possibility), we obtain two possibility functions, one for $P_{t,L} \sim MRP\left(k,\overline{P},\sigma_L\right)$ and one for $P_{t,U} \sim MRP\left(k,\overline{P},\sigma_U\right)$ (Figure 6).

The uncertainity of the metal price, at point *t*, is further modeled by a type-2 fuzzy set where possibility function for $P_{t,L}$ corresponds to lower membership function and possibility function for $P_{t,U}$ to upper membership function, respectively.

The concept of type-2 fuzzy set was introduced by Zadeh (1975) as an extension of the concept of an ordinary fuzzy set (called a type-1 fuzzy set). For more details see (Liu, 2008; Karnik and Mendel, 2001; Dinagar and Anbalagan, 2011; Nasab and Malkhalifeh, 2010).

A type-2 fuzzy set, denoted $\overset{\approx}{A}$, is characterized by a type-2 membership function $\mu_{\overset{\approx}{A}}(x,u)$, i.e., $\overset{\approx}{A} = \left\{(x,u), \mu_{\overset{\approx}{A}}(x,u) \middle| \forall x \in X, \forall u \in J_x \subseteq [0,1]\right\}$, in which $0 \le \mu_{\overset{\approx}{A}}(x,u) \le 1$, J_x is the primary membership, which is domain of the secondary membership function. The amplitude of a secondary membership function is called the secondary grade. When the secondary membership functions are type-1 interval sets, we call the type-2 set an interval type-2 set.

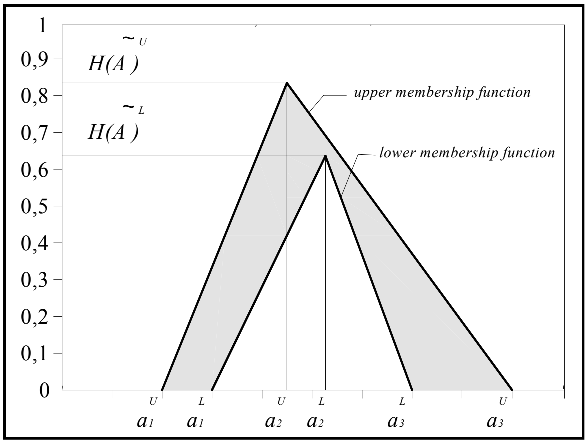

Figure 7 - A triangular interval type-2 fuzzy set

The interval type-2 $\overset{\approx}{A}$, can be represented as $\overset{\approx}{A} = \left(\tilde{A}^U, \tilde{A}^L\right)$ where $\tilde{A}^U, \tilde{A}^L$ are upper membership function and lower membership function, respectively. Note that $\tilde{A}^U$ and $\tilde{A}^L$ are fuzzy type-1 sets. The triangular interval type-2 fuzzy set $\overset{\approx}{A}$ can be represented as follows: $\overset{\approx}{A} = \left(\tilde{A}^U, \tilde{A}^L\right) = \left(\left(a_1^U, a_2^U, a_3^U; H\left(\tilde{A}^U\right)\right), \left(a_1^L, a_2^L, a_3^L; H\left(\tilde{A}^L\right)\right)\right)$

where $a_1^U \le a_2^U \le a_3^U$; $a_1^L \le a_2^L \le a_3^L$; $a_1^U \le a_1^L$ and $a_3^L \le a_4^U$, i.e., $\tilde{A}^L \subset \tilde{A}^U$. $H\left(\tilde{A}^U\right)$ and $H\left(\tilde{A}^L\right)$, denote the heights of $\tilde{A}^U$ and $\tilde{A}^L$, respectively, $H\left(\tilde{A}^U\right) \in [0,1]$ and $H\left(\tilde{A}^L\right) \in [0,1]$. Figure 7 represents the upper triangular membership function $\tilde{A}^U$ and the lower triangular membership function $\tilde{A}^L$ of the interval type-2 fuzzy set $\approx\!\!A$.

The uncertainty of the metal price, at point (year) *t*, is now expressed as the triangular interval type-2 fuzzy number, $\overset{\approx}{P}_1 = \left(\tilde{P}_1^U, \tilde{P}_1^L\right)$, $\overset{\approx}{P}_2 = \left(\tilde{P}_2^U, \tilde{P}_2^L\right)$, ..., $\overset{\approx}{P}_T = \left(\tilde{P}_T^U, \tilde{P}_T^L\right)$, (Figure 8).

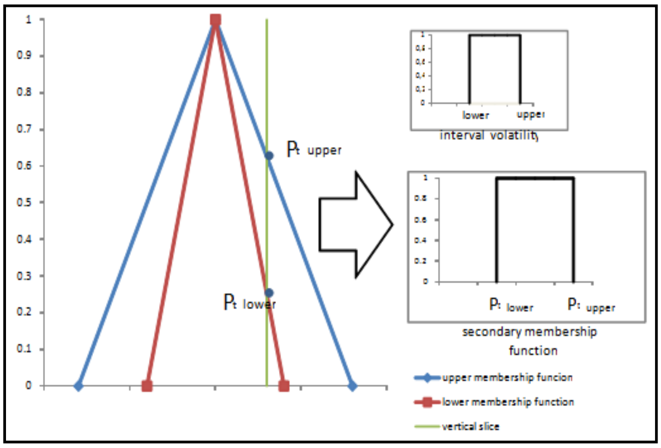

Figure 8 - A triangular interval type-2 fuzzy number of metal price

An important concept related to the application of interval type-2 fuzzy numbers is defuzzification, which converts a interval type-2 fuzzy number into a crisp value. Such transformation is not unique because different methods are possible. In this paper we applied the following transformation.

Karnik and Mendel have developed two iterative algorithms (known as the KM Algorithms) for defuzzification of the interval type-2 fuzzy sets (Karnik and Mendel, 2001). These algorithms are monotonically and super-exponentially convergent (Mendel and Liu, 2007).

The KM Algorithms can be explained as follows.

The c_r (right) determination

1) Assume that the pre-computed c_r^i are arranged in ascending order, i.e., $c_r^1 \le c_r^2 \le ... \le c_r^M$, where *M* denotes number of vertical slices.

2) Make initialization of θ_r^i, through following equation:

$$\theta_r^i = \frac{1}{2}\left[\underline{\mu}\left(c_r^i\right) + \overline{\mu}\left(c_r^i\right)\right], \quad i = 1,2,...,M \tag{19}$$

3) Compute c_r as:

$$c_r = c\left(\theta_r^1, \theta_r^2, ..., \theta_r^M\right) = \frac{\sum_{i=1}^{M} c_r^i \cdot \theta_r^i}{\sum_{i=1}^{M} \theta_r^i} \tag{20}$$

and let $c_r' \equiv c_r$.

4) Find $R\left(1 \le R \le M-1\right)$ such that $c_r^R \le c_r' \le c_r^{R+1}$.

5) Compute c_r as:

$$c_r = \frac{\sum_{i=1}^{R} c_r^i \cdot \underline{\mu}\left(c_r^1\right) + \sum_{i=R+1}^{M} c_r^i \cdot \overline{\mu}\left(c_r^1\right)}{\sum_{i=1}^{R} \underline{\mu}\left(c_r^1\right) + \sum_{i=R+1}^{M} \overline{\mu}\left(c_r^1\right)} \tag{21}$$

and let $c_r'' \equiv c_r$.

6) Check, if $c_r'' = c_r$ than stop and set $c_r'' \equiv c_r$. If not, set $c_r' = c_r''$ and return to Step 4.

The c_l (left) determination

1) Replace c_r^i by c_l^i.

2) Make initialization of θ_l^i through equation (19).

3) Compute c_l through equation (20).

4) Find $L\left(1 \le L \le M-1\right)$ such that $c_l^L \le c_l' \le c_l^{L+1}$.

5) Compute c_l as:

$$c_l = \frac{\sum_{i=1}^{L} c_l^i \cdot \overline{\mu}\left(c_l^1\right) + \sum_{i=L+1}^{M} c_l^i \cdot \underline{\mu}\left(c_l^1\right)}{\sum_{i=1}^{L} \overline{\mu}\left(c_l^1\right) + \sum_{i=L+1}^{M} \underline{\mu}\left(c_l^1\right)} \tag{22}$$

and let $c_l'' \equiv c_l$.

6) Check, if $c_l'' = c_l$ than stop and set $c_l'' \equiv c_l$. If not, set $c_l' = c_l''$ and return to Step 4.

The defuzzified output of an interval type-2 fuzzy set is simply the average of c_l and c_r, i.e.,

$$c = \frac{1}{2}\left(c_l + c_r\right)$$

The defuzzified output of an interval type-2 fuzzy number of metal price is simply the average of $P_{l,(t)}$ and $P_{r,(t)}$, $t = 1, 2, ..., T$ i.e., $P_{(t)} = \frac{1}{2}\left(P_{l,(t)} + P_{r,(t)}\right)$ $t = 1, 2, ..., T$.

Algorithm of quantification of uncertainty of metal price is represented in Figure 9.

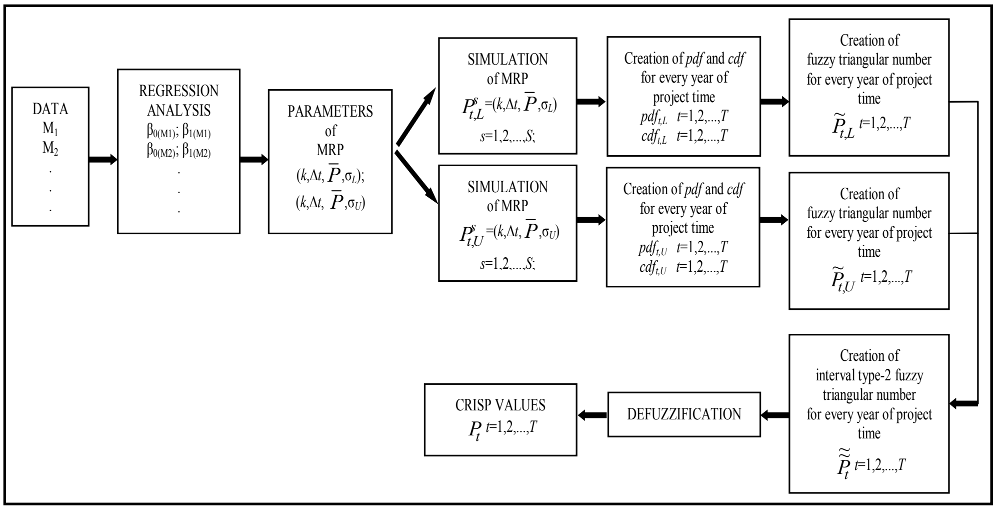

Figure 9 - Flow chart of quantification of uncertainty related to metal price

3. CONCLUSION

The stochastic behavior of metal price has important implications for the valuation of natural resource project related to the prices of those commodities. The Mean reversion process approach to forecast future prices of metal is going support both in the scholar community and in the practice. Validity of the parameters of the Mean reversion process directly depends on source of information. Selection of real source plays a key role in the process of estimation of parameters. To avoid this problem we propose the regression analysis for every source of historical data of metal price. Since regress observation has been done we obtained the parameters of MRP as interval numbers. The speed of mean reversion and the long-run equilibrium metal price are fixed and they are defined according to interval arithmetic while volatility stays as interval with uniform variation within it. On the base of results of MRP simulation we obtain two probability and two cumulative distribution functions (upper and lower) for every year of the project time. According to way of transformation (from probability to possibility) and definition of fuzzy triangular number we create interval type-2 fuzzy triangular numbers representing the prices of metal for every year of the project time. Interval type-2 fuzzy triangular numbers are defuzzified and converted into a crisp values by using Karnik and Mendel two iterative algorithms. Forecasting future values of metal prices is crucial component in the evaluation of mining project. Model developed in this paper quantifies uncertainty of the metal market and helps decision makers to involve it into process of the mining project evaluation. This approach is not limited only for this purpose, it can be used to forecast, for example, the future values of costs of production because many authors use MRP to describe fluctuations of oil and electricity prices.

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