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Professional paper

1D ADJUSTMENT OF THE GEODETIC NETWORK AT THE SCHOOL MINE "CRVENI BREG" ON AVALA

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Abstract: The objective of this paper is to present the procedure for obtaining the most probable values of height differences through one-dimensional adjustment of the geodetic network of the school mine "Crveni Breg" on Avala. Since the terrain is not connected to the national levelling network, the first step involves determining adjusted height differences. Once the initial point is linked to a national benchmark, accurate elevations of all points within the school polygon will be established. The survey was conducted using the geometric levelling method from the middle (''forward-backward''), and the results were processed by conditional adjustment. Accuracy assessment indicated that the obtained standard deviation corresponds to technical levelling of increased precision, confirming the quality of the fieldwork and measurements.

Keywords: conditional adjustment, geodetic network, accuracy assessment.

1 INTRODUCTION

In contemporary research practice, numerous scientific disciplines are based on measuring specific quantities, their analysis, and interpretation. Throughout the process of measuring, we obtain values/results with lower or higher accuracy compared to the true value, which cannot be directly reached by measurement. By repeatedly measuring the same value, different results are usually obtained, leading to the conclusion that all measurements are burdened with errors. Some errors can be eliminated through measurement procedures by subsequently introducing corrections, however, random errors remain and are an integral part of measurement results [Ganić, 2008; Džeparovski, 1995; Ghilani, 2010].

In order to minimize random errors in measurement results and to achieve their closer approximation to the true values, as well as to the computed parameters derived from these results, it is necessary to ensure measurement redundancy. This is accomplished

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not only by performing an adequate number of repeated observations of the same quantities, but also by observing a greater number of quantities than strictly required for the unique determination of the unknowns (in the case of mining surveys, these unknowns are the coordinates and/or elevations of points) [Ganić, 2008; Džeparovski, 1995; Ghilani, 2010].

The indeterminacy of such a mathematical system is resolved by applying rigorous adjustment methods, which are based on the condition that the sum of the squares of the corrections to the observed values (v_i) is minimized (equations 1a and 1b) [Ganić, 2008; Džeparovski, 1995; Ghilani, 2010].

$$\sum_{i=1}^{n}(i = 1)^{n} v_i \, v_i = min \tag{1a}$$

respectively

$$\sum_{i=1}^{n} p_i v_i \, v_i = min \tag{1b}$$

depending on whether the measurements are of equal or varying precision

Given the stated minimum condition, the results of repeated measurements of the same values are subjected to direct adjustment, thereby yielding the most probable values of the observed quantities. Subsequently, since a redundant number of observations are available, all most probable values of the measured quantities are incorporated into the procedure of indirect or conditional adjustment, with the aim of obtaining the most probable values of the unknown parameters. Namely, the coordinates and/or elevations of points [Ganić, 2008; Džeparovski, 1995; Ghilani, 2010].

During mining surveying, we observe angles, distances, and height differences, and through their processing and further calculations, the coordinates and elevations of points are derived. These serve as the basis for solving a wide range of tasks encountered during the exploitation of mineral resources, as well as for the graphical representation of terrain or the staking out of designed structures.

The polygon of the school mine "Crveni Breg" on Avala, consisting of trigonometric points and benchmarks on the surface terrain, as well as all points of the underground traverse in the lower gallery, is not vertically connected to the national levelling network. Consequently, all these points possess only approximate local elevations. For this reason, measurements of height differences were carried out within the surface polygon, and their processing was performed using conditional adjustment to obtain the most probable values of the height differences, as a first step. This paper presents the procedure of one-dimensional conditional adjustment of the polygon of the school mine "Crveni Breg" on Avala [Đurđev, 2023].

In the future, once a point of the polygon is connected to the national levelling network and its true elevation is determined, the true elevations of all other points in the polygon will subsequently be calculated based on the most probable values of height differences.

2 FIELD MEASUREMENTS OF HEIGHT DIFFERENCES AND MEASUREMENT RESULT

The school mine "Crveni Breg" on Avala dates to Roman times, and traces of their exploitation are still visible in certain parts of the mine. "Crveni Breg" is a lead and silver mine, officially opened in 1886/1887 and officially closed in 1953. Today, the "Crveni Breg" mine is used for educational purposes.

The geodetic basis of the polygon of the school mine on the surface terrain in front of the lower gallery consists of six points: four trigonometric points (101–104) and two benchmarks (Rp and Rz), as shown in Figure 2.1.

Figure 2.1 Research location: School Mine "Crveni Breg" on Avala and the Geodetic Basis of the School Mine (Source: Google maps)

The geodetic basis of the school mine on Avala is not strictly connected to the national levelling network, and therefore the elevations of its points are of approximate character. For this reason, conditional adjustment of the measured height differences within the network will be applied, yielding their most probable values. The elevations of the network points will not be calculated at this stage, however, once the network is connected to the national benchmarks, the most probable values of the height differences obtained in this way will be used to calculate the most probable elevations of all points in the network [Đurđev, 2023].

Within the geodetic network, nine height differences were measured, thereby ensuring redundant observations. A sketch showing the mentioned control points and the measured height differences (Δh_i) of the levelling lines is presented in Figure 2.2. [Đurđev, 2023].

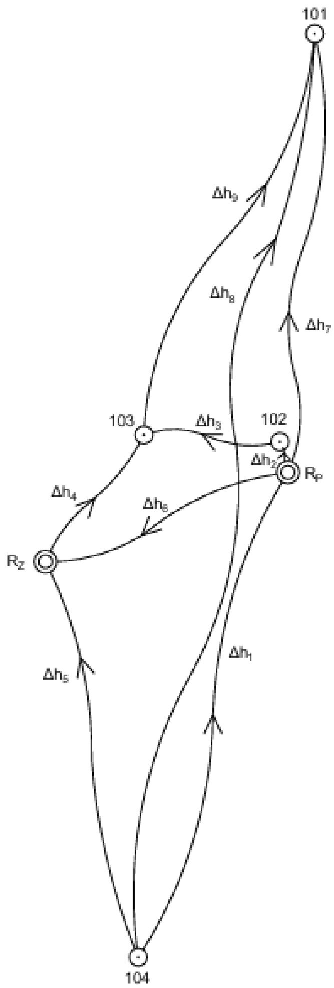

Figure 2.2 Sketch of the Levelling Network (Approximate scale) [Đurđev, 2023]

The measurements were carried out using a digital levelling instrument "Leica Sprinter 150M," whose standard deviation at a distance of 30 m amounts to ±0.6 mm [Leica

Geosystems]. Supporting equipment was used, including aluminum levelling bars and iron shoes for the stabilization of reference points.

The height differences were measured using the method of geometric levelling from the midpoint (backsight–foresight). The readings on the staffs were taken according to the BFFB principle (back–front–front–back) with a change in instrument height. In this way, four independent measurements were ensured for each levelling line in the network.

Table 2.1 presents the calculated average values of the measured height differences, as well as the achieved and permissible maximum deviations of the height differences.

The average achieved deviation of the measured height differences throughout the entire levelling network is 1.4 mm. The permissible deviations of the measured height differences were calculated according to the formula for technical levelling of higher accuracy on favorable terrain [Mihajlović, 1982], while the achieved deviations are significantly smaller than the permissible ones

Table 2.1 Calculated Average Values of the Measured Height Differences and Achieved Standard Deviations [Đurđev, 2023]

Traverse number	from	to	Discrepancy Forward (m)		Discrepancy Backward (m)		Average (m)		Deviations (mm)	
			Δh_p	S_p	$-\Delta h_z$	S_z	Δh	S	Ach[2].	Perm. [3]
1	104	R_p	0.8800	113	0.8785	112	0.87925	112.25	1.5	6.7
2	R_p	102	0.5400	7	0.5405	7	0.54025	7	0.5	1.7
3	102	103	5.2380	30	5.2400	30	5.239	30	2	3.5
4	R_z	103	5.2715	35	5.2720	36	5.27175	35.5	0.5	3.8
5	R_z	104	-1.3830	88	-1.3850	88	-1.384	88	2	5.9
6	R_z	R_p	-0.5075	32	-0.5065	32	-0.507	32	1	3.6
7	R_p	101	14.6675	122	14.6665	121	14.667	121.5	1	7.0
8	104	101	15.5485	207	15.5465	209	15.5475	208	2	9.2
9	103	101	8.8900	112	8.8875	112	8.88875	112	2.5	6.7

3 DATA PROCESSING AND MOST PROBABLE VALUES OF HEIGHT DIFFERENCES

The levelling network of the school mine on Avala was adjusted by the method of conditional adjustment, treated as a free levelling network in the local system. Since the elevations of all six points are unknown, a local elevation was adopted for benchmark Rz, while the elevations of the remaining five points of the network remain as unknown. Given that nine height differences were measured, this implies the availability of four redundant observations, and for each redundant observation it is necessary to establish one independent conditional equation [Đurđev, 2023].

[2]Achieved deviation

[3]Permissible deviation

The final conditional equations are of the form

$$-\Delta h'_1 + \Delta h'_5 - \Delta h'_6 = 0 \quad (2)$$

$$-\Delta h'_2 - \Delta h'_3 + \Delta h'_4 + \Delta h'_6 = 0 \quad (3)$$

$$\Delta h'_4 + \Delta h'_5 - \Delta h'_8 + \Delta h'_9 = 0 \quad (4)$$

$$-\Delta h'_1 - \Delta h'_7 + \Delta h'_8 = 0 \quad (5)$$

The conditional adjustment of the levelling network was carried out by applying matrix solution techniques in the program "Microsoft Excel". For the purposes of adjustment using matrix calculations, based on the final conditional equations, the following were formed:

- Coefficient matrix of the conditional equations *A*:

$$A = \begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

- Vector of independent terms *ω*:

$$\omega = \begin{bmatrix} -0.00225 \\ -0.0005 \\ -0.003 \\ 0.00125 \end{bmatrix}$$

- Weight matrix *P*:

$$P = \begin{bmatrix} 8.9087 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 142.85714 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 33.3333 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 28.1690 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 11.3636 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 31.2500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8.2304 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.8077 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8.9286 \end{bmatrix}$$

Based on these, the following values are calculated according to the equations:

- Coefficient matrix of the normal equations *N:*

$$N = A^T \cdot P^{-1} \cdot A \tag{6}$$

Inverse matrix of the normal equation coefficients N^{-1}

- Vector of correlates *k*:

$$k = -N^{-1} \cdot \omega \tag{7}$$

- Correction vector *v:*

$$Nv = P^{-1} \cdot A \cdot k \tag{8}$$

By adding the calculated corrections to the measured height differences, their most probable values are obtained, which satisfy the imposed conditional equations:

$$\Delta h'_1 = \Delta h_1 + v_1 = +0.8784 \text{ m } (104 - \mathrm{R_p})$$

$$\Delta h'_2 = \Delta h_2 + v_2 = +0.5402 \text{ m } (\mathrm{R_p} - 102)$$

$$\Delta h'_3 = \Delta h_3 + v_3 = +5.2387 \text{ m } (102 - 103)$$

$$\Delta h'_4 = \Delta h_4 + v_4 = +5.2721 \text{ m } (\mathrm{R_z} - 103)$$

$$\Delta h'_5 = \Delta h_5 + v_5 = -1.3852 \text{ m } (\mathrm{R_z} - 104)$$

$$\Delta h'_6 = \Delta h_6 + v_6 = -0.5068 \text{ m } (R_z - R_p)$$

$$\Delta h'_7 = \Delta h_7 + v_7 = +14.6677 \text{ m } (R_p - 101)$$

$$\Delta h'_8 = \Delta h_8 + v_8 = +15.5461 \text{ m } (101 - 104)$$

$$\Delta h'_9 = \Delta h_9 + v_9 = +8.8887 \text{ m } (103 - 101)$$

The reference standard deviation of unit weight σ_o amounts to:

$$\sigma_o = \sqrt{\frac{v^T \cdot P \cdot v}{n - r}} = \pm 3.32 mm/km \tag{9}$$

where the difference *n* - *r* represents the number of redundant measurements in the network.

The standard deviations of the adjusted height differences are calculated according to the equation:

$$\sigma_{\Delta h_{i'}} = \sigma_o \cdot \sqrt{Q_{h_i} \cdot Q_{h_i}} \tag{10}$$

Where:

$$Q_{h_i} = P^{-1} \cdot P^{-1} \cdot A \cdot N^{-1} \cdot A^T \cdot P^{-1} \tag{11}$$

And they amount to:

$$\sigma_{\Delta h_1} = \pm 0.72 \text{ mm}$$
$$\sigma_{\Delta h_2} = \pm 0.27 \text{ mm}$$
$$\sigma_{\Delta h_3} = \pm 0.46 \text{ mm}$$
$$\sigma_{\Delta h_4} = \pm 0.49 \text{ mm}$$
$$\sigma_{\Delta h_5} = \pm 0.72 \text{ mm}$$
$$\sigma_{\Delta h_6} = \pm 0.46 \text{ mm}$$
$$\sigma_{\Delta h_7} = \pm 0.76 \text{ mm}$$
$$\sigma_{\Delta h_8} = \pm 0.90 \text{ mm}$$
$$\sigma_{\Delta h_9} = \pm 0.77 \text{ mm}$$

4 CONCLUSION

In this study, the procedure for calculating the most probable values of the measured height differences in the polygon of the school mine "Crveni Breg" on Avala, in front of the lower adit, is presented. The calculation of the most probable values of the measured height differences within the polygon was carried out using the method of conditional adjustment. The polygon was adjusted as a free local network, since in the future it will be connected to the national levelling network, at which time the points will obtain their official elevations.

The accuracy assessment after adjustment showed that the standard deviation of the measured height differences in the network is ±3.32 mm/km. This deviation is smaller than the permissible deviation in a levelling network of increased technical accuracy, which amounts to ±5 mm/km, thus indicating the high quality of the fieldwork and measurements performed. Finally, the most probable values of the height differences were computed, satisfying the imposed mathematical conditions, as well as their standard deviations, which range between ±0.27 mm and ±0.90 mm.

In future work, when the school polygon is connected to the national benchmarks, it will be possible to calculate the true elevations of all points forming the polygon. Moreover, the standard deviations of the elevations of the polygon points, given the small standard deviations of the adjusted height differences, will predominantly depend on the standard deviation of the connected point from the national network, and within the entire network will approximately equal the standard deviation of that connected point.

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