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Original scientific paper

METHODOLOGY FOR DIMENSIONING THE HOISTING SHAFT USING MATHEMATICAL AND GRAPHICAL METHODS

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Abstract: The opening of deep ore bodies is most rapidly accomplished using vertical shafts. Shaft construction is the most economically demanding process, so despite providing the fastest access to deep ore bodies, the high capital costs involved in construction mean that determining the cross-sectional dimensions of the shaft is a complex and very important step in the design of the shaft and the entire underground mine. An inadequate cross-sectional area can have negative impacts: a small area limits the mine's capacity, while a large area increases construction costs and affects shaft stability. In modern mining practice, shafts are mostly constructed with a circular cross-section, whose area depends on a single parameter - shaft diameter. The choice of area, or more specifically the minimum shaft diameter, is generally determined empirically using mathematical or graphical methods. If there are indications that production capacity might increase or that greater volumes of air will be required, adjustments are made, and the shaft diameter is increased. This paper will present an extended mathematical model and calculations to accurately determine the dimensions of skips for ore hoisting, followed by a graphical method for determining the shaft diameter based on the mathematical calculation.

Keywords: underground exploitation, dimensioning, hoisting shaft, ore hoisting

1 INTRODUCTION

Shafts are vertical mining structures characterized by a relatively small cross-section compared to their length (depth). These structures may have a direct connection to the surface or not (blind shaft). The high cost of construction also necessitates the multifunctionality of underground spaces. In addition to their primary purpose—by which shafts are named (hoisting shaft, ventilation shaft, service shaft, drainage shaft, backfill shaft)—they are also used for air circulation, employee movement, transport,

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maintenance, etc. Depending on their importance, shafts can be classified as main or auxiliary, central, or of lesser significance based on their location and purpose. (Antunović Kobliška, 1973; Hartman Howard, 1992; Torbica & Lapčević, 2020).

As the method of opening that provides the fastest access to deep ore bodies, the vertical shaft represents one of the most important elements of underground mine infrastructure, as it enables direct access to mineral deposits at great depths.

The shaft used for mine development is initially assigned to the primary purpose of ore hoisting and becomes a hoisting shaft. The hoisting shaft can be classified according to the type of hoisting vessel used (Music, 2014):

- Skip shaft – This type may have a single skip with a counterweight or two/four coupled skips. These shafts are constructed in mines with high production rates, and therefore, one or more additional auxiliary shafts must be built alongside for transporting personnel, materials, etc.
- Skip-cage shaft – Constructed in mines with low to medium production. In addition to ore transportation, this shaft also takes on the role of an auxiliary shaft, serving for servicing the mine. This type of shaft comes in several combinations, such as two skips and a cage with a counterweight, two skips and two cages, etc.
- Cage shaft – Used in mines with low production. Ore is transported in mine carts, which are brought to the surface using cages. The shaft may have two coupled cages or one cage with a counterweight. Nowadays, this method of hoisting is rarely used, as very few mines still rely on shaft hoisting in combination with rail transport.

With skips and cages using counterweights, ore can be hoisted from multiple levels, whereas with coupled conveyances, ore can only be hoisted from a single level.

As previously mentioned, selecting the optimal dimensions of the shaft's cross-section is crucial, since these dimensions determine the scope of required work, capacity limitations, and the type of equipment that will be used throughout the entire lifespan of the mine. This means that the planned dimensions must meet operational requirements over decades of exploitation, so long-term production planning must be taken seriously and carried out for the entire life of mine to avoid unnecessary expenses.

2 DEFINING DIMENSIONS OF VERTICAL SHAFTS

When designing mining openings, the most important factor is the selection of the shape and size of the cross-section.

The shape of the shaft's cross-section is determined based on the physical and mechanical properties of the rock, the shaft's intended purpose, and its operational lifespan. In addition to these factors, the choice of cross-sectional shape is also influenced by construction costs, construction speed, and maintenance costs (Musić, 2014).

The type of rock and the expected lifespan of the shaft determine the type of support structure, while the support material affects the choice of cross-sectional shape.

The most common shaft cross-section shapes are circular and rectangular, with elliptical and combined shapes used less frequently (Jovanović, 1982).

The size of the shaft's cross-section depends on its purpose. For a hoisting shaft with a given capacity, the decisive factors are the dimensions of the hoisting vessels, their number, and arrangement (Jovanović, 1982).

In addition, consideration must be given to the auxiliary equipment installed in the shaft, as well as the requirements for a specific volume and velocity of air passing through the shaft.

In this paper, a calculation of the skip dimensions will be carried out, and based on that, the sizing of the shaft's cross-sectional area will be performed.

2.1 Calculation of skip dimensions

The size, or dimensions, of the skip are determined by the required payload per trip, which is influenced by the planned mine capacity. Achieving the required capacity and the shaft depth are the starting parameters for this calculation. Detailed calculations can be found in literature (Pavlović, 1963; Grujić, 1999).

Necessary hourly capacity (Q_h) on hoisting operations can be calculated as follows:

$$Q_h = \frac{k \cdot Q_{year}}{d \cdot h_d} \tag{1}$$

Where:

Q_{year} – yearly capacity (t per year),
k – coefficient of uneven production (1.1 – 1.3),
d – number of working days in a year,
h_d – effective working hours in a day.

To perform the calculation, it is necessary to know the number of effective working hours during one shift. This time is obtained by subtracting the time needed for transporting personnel and shaft inspection from the total duration of the shift (8 or 12 hours).

The time required for transporting personnel is calculated based on:

$$t_{per} = \frac{h}{v_n} \cdot \frac{N}{n} + \frac{N}{n} \cdot t_m \quad (2)$$

Where:

h – shaft depth (m),
N – number of workers in one shift,
n – number of workers in a skip,
t_m - the maneuvering time when workers enter the skip (approximately 3 seconds per worker),
v_n - the speed of the vessel during the transportation of workers (m/s).

The maximum allowed speed of travel (m/s) is calculated using the following formula:

$$v_{max} = k_i \cdot \sqrt{\frac{2 \cdot p \cdot q}{p+q} \cdot h}, \ m/s \quad (3)$$

Where:

k_i – speed utilization coefficient,
p - hoisting vessel acceleration (m/s^2),
q - hoisting vessel deceleration (m/s^2),
h – shaft depth (m).

According to (Službeni list SFRJ, 1992):

- The maximum allowed value of the speed utilization coefficient is 0.6,
- The maximum allowed acceleration and deceleration is 1 m/s^2,
- The maximum allowed speed for transporting people is 14 m/s, and the maximum allowed speed for transporting material is 20 m/s.

These parameters may vary in different countries legislatively.

Generally, when calculating the maximum speed, the values are adopted by considering the following relationships:

$$p \approx q \geq 0{,}6, \frac{m}{s^2} \quad (4)$$

$$k_i \cdot \sqrt{p} \geq 0{,}5 \quad (5)$$

It is recommended (or it can be adopted) that the travel speed be 20% lower than the maximum travel speed, i.e., $v_n = 0.8 \cdot v_{max}$.

The shaft revision must be carried out before the start of each shift. The duration of the shaft revision can be calculated using the following formula:

$$t_r = 2 \cdot (\frac{h}{0,5} + 800), \text{ s} \quad (6)$$

The number of trips per hour is calculated using the formula:

$$n_h = \frac{3600}{T} \quad (7)$$

Where T represents the total duration of one trip (s) and is calculated as the sum of the travel time (t) and the maneuvering time (t_m):

$$T = t + t_m, \text{ s} \quad (8)$$

The travel time is determined using the following formula:

$$t = \frac{h}{v} + 25, \text{ s} \quad (9)$$

The maneuvering time for ore hoisting using skips is approximately 10 seconds.

The payload of one trip is calculated for two cases:

- In the case of using two coupled hoisting vessels:

$$Q = \frac{Q_h}{n_h}, \text{ t} \quad (10)$$

- In the case of using hoisting vessels with a counterweight:

$$Q = 2\frac{Q_h}{n_h}, \text{ t} \quad (11)$$

The required volume of the skip is calculated:

$$V_S = \frac{Q}{\gamma}, m^3 \quad (12)$$

Where:

γ - bulk density of crushed ore (t/m^3).

The dimensions of the shaft were adopted, from literature (Music, 2014; Torbica, 2020), based on the volume of the skip (shown in Table 1).

Based on two conditions, or two equations, the exact dimensions of the skip can be calculated.

The first condition is the volume of the skip, which is equal to:

$$V_s = P_p \cdot h_s,\ m^3 \tag{13}$$

Where:

P_p - the cross-sectional area of the skip, m^2,

h_s - the height of the skip, m.

Skips can have a rectangular (14) or square cross-section (15), so the areas are:

$$P_p = a \cdot b,\ m^2 \tag{14}$$

$$P_p = a^2,\ m^2 \tag{15}$$

For a rectangular (a, b) or square cross-section of the skip (a, a), the diagonal can be determined using the Pythagorean theorem.

$$d^2 = a^2 + a^2 = 2a^2 \tag{16}$$

$$d = a\sqrt{2},\ m \tag{17}$$

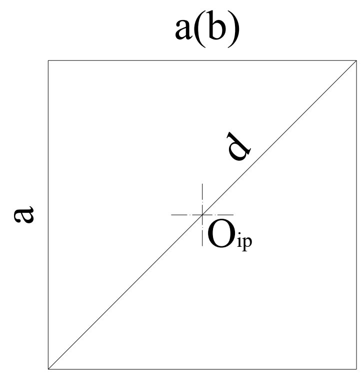

Figure 1 Diagram for determining the diagonal of the hoisting vessel with a rectangular cross-section (O_{ip} – center of gravity of the hoisting vessel)

The second condition, or the second equation, is that the height of the skip (h_s) is equal to twice the value of the diagonal of the skip's cross-section.

$$h_s = 2d,\ m \tag{18}$$

That is, based on formula (17), the value of the diagonal is

$$h_s = 2d = 2a\sqrt{2},\ m \tag{19}$$

This value for the height of the skip can be substituted into the equation for the skip's volume, yielding:

$$V_s = a^2 \cdot h_s = a^2 \cdot 2a\sqrt{2} = 2a^3\sqrt{2},\ m^3 \tag{20}$$

Considering that the volume of the skip is already known, the side length of the square cross-section of the skip can be calculated from this equation.

$$a = \sqrt[3]{\frac{0{,}5V_s}{\sqrt{2}}},\ m \tag{21}$$

When this value is known, the cross-sectional area and the diagonal of the cross-section can be calculated:

$$P_p = a^2,\ m^2$$
$$\mathrm{d} = a\sqrt{2},\ m. \tag{22}$$

From these two equations, the dimensions of the rectangle (a, b) can be obtained if a rectangular cross-section of the skip is required:

$$P_p = a \cdot b,\ m^2$$
$$d = \sqrt{a^2 + b^2},\ m. \tag{23}$$

Where P_p and d are known values.

Therefore, even in the case of a required rectangular cross-section, the calculation is initially performed as if a square cross-section is used, and then the exact dimensions of the rectangle in the cross-section are retroactively calculated. From Table 1, the ratio of the longer to the shorter side of the rectangle is 1:0.9 to 1:0.8, so this should be considered when selecting the sides.

2.2 Graphical determination of the shaft dimensions

The dimensions of the cross-section are determined graphically by plotting (at a scale of 1:1) the selected hoisting vessels, considering the minimum required distances according to the relevant standards.

The mathematically and graphically determined elements are sufficient for constructing the appearance of the cross-section.

One of the possible cross-sections with two hoisting vessels is shown in Figure 2.

The labels from Figure 2 are:

- a – length of the hoisting vessel, m,
- b – width of the hoisting vessel, m,
- c – distance between hoisting vessels, m,
- d – eccentricity, m,
- e – distance from the outermost point of the hoisting vessel to the side of the structure (support), m,
- f – distance from the edge of the hoisting vessel to the support, m,
- m and n – dimensions of the guide, m,
- D – clear diameter of the shaft, m,
- C – center of the shaft.

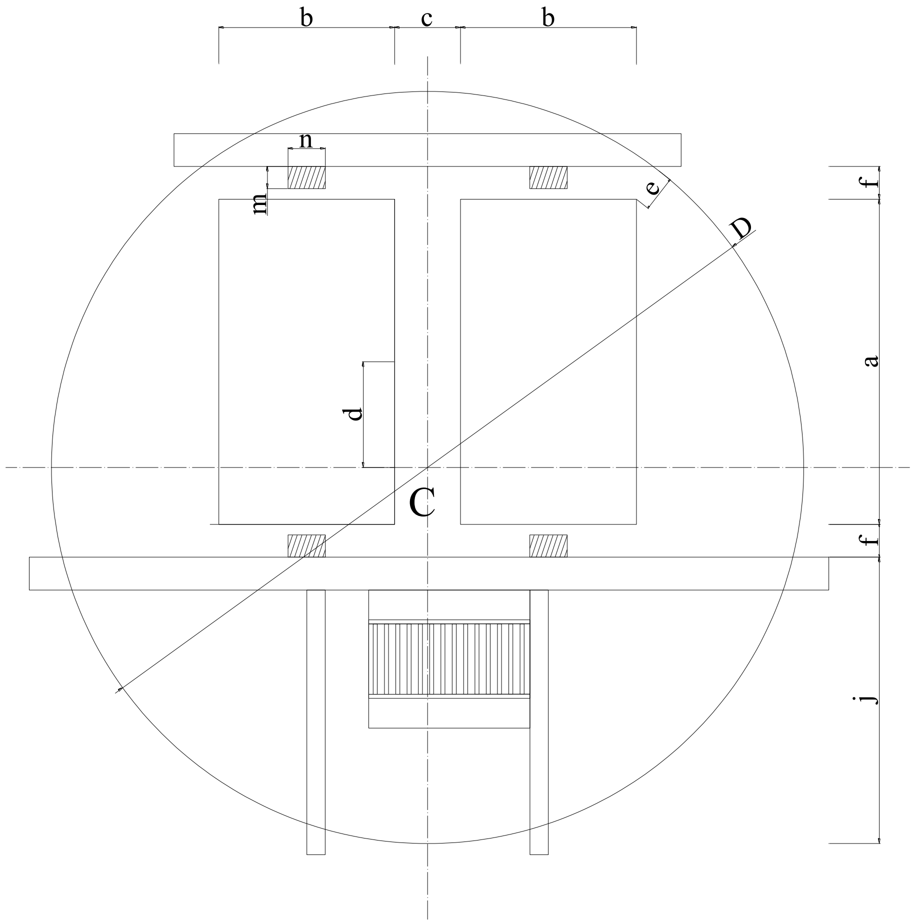

Figure 2 Shaft with a circular cross-section (two skips, pass-through section)

The listed values are variable and depend on the capacity and dimensions of the hoisting vessels. As an illustration of this statement, data is provided in Table 1, where the change in these values is shown depending on the skip's capacity.

Table 1 Adopted distances and dimensions for specific skip volumes (Musić, 2014

Skip (m^3)	D (m)	a (m)	b (m)	c (m)	d (m)	e (m)	f (m)
1	2,96	1	0,9	0,2	0,39	0,14	0,17
2	3,38	1,3	1,1	0,2	0,34	0,14	0,18
3	4,15	1,8	1,45	0,25	0,23	0,14	0,18

Since this paper presents the complete calculation up to the precise dimensions of the skips, the remaining dimensions (c, d, e, f, ...) are adopted from the corresponding tables.

Graphical determination is based on the same principle for both square and rectangular cross-sections of the skip. The design begins by drawing the skip with side a, as shown in Figure 3. In the cross-section, the diagonal contains the central point of the skip (the center of gravity), O_{ip}.

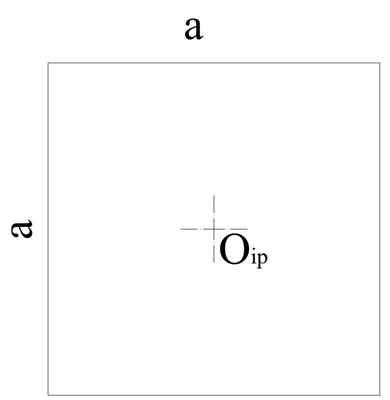

Figure 3 Skip with side a

From the right side of the skip, an auxiliary line is drawn at a distance of *c/2*, parallel to the right side of the skip. At a distance of *d* from the projection of point O_{ip} onto this line, the center of the shaft cross-section is located, as shown in Figure 4.

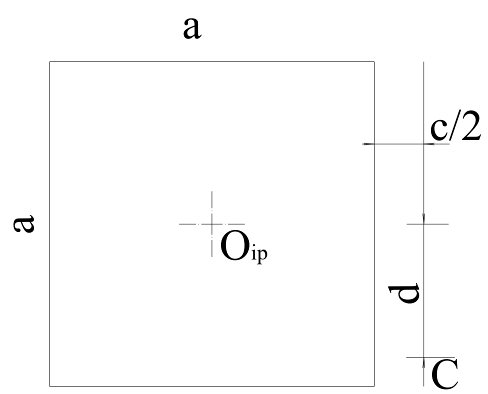

Figure 4 Determination of the shaft center

At a distance of *c* from the right side of the skip, a second skip/counterweight (usually of the same size) is drawn. Center *C* is connected to the upper left corner of the skip, and then extended by a distance e, which gives the radius of the shaft, as shown in Figure 5.

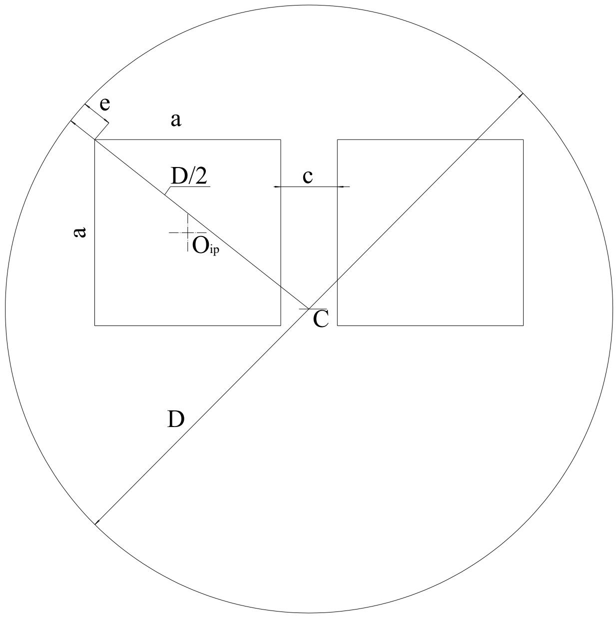

Figure 5 Determining the radius of the shaft

Finally, at a distance of *f* from the lower edge of the skip, the main and auxiliary transverse steel supports are projected. In the remaining part of the cross-section, the departments are organized: a passage section with a ladder and rest areas, as well as sections for necessary pipelines, cables, and other equipment (Figure 6).

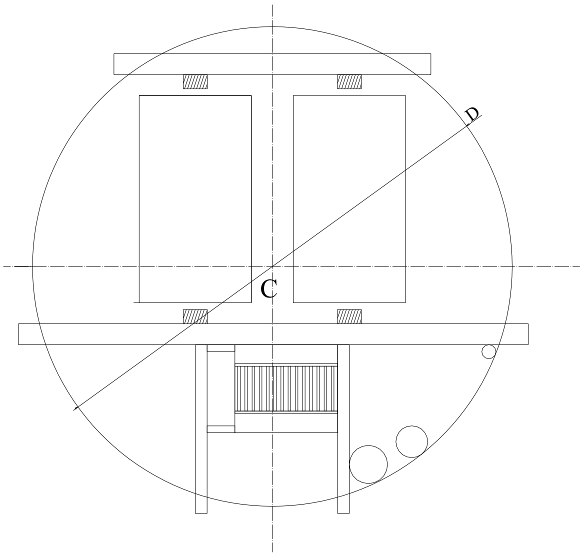

Figure 6 Construction of the entire shaft

As already mentioned, skips are custom-made. The dimensions of the shafts depend on the dimensions of the skips, which are determined by the capacity of the mine and the shaft depth. When constructing the shafts, it is also necessary to determine the thickness of the required support structure (lining) in order to calculate the sufficient area of the excavation cross-section, whose diameter (D_{isk}) is equal to the sum of the diameter of the clear cross-section (D_{sv}) (shown in Figure 6) and twice the thickness of the support structure (Dp), as shown in Figure 7.

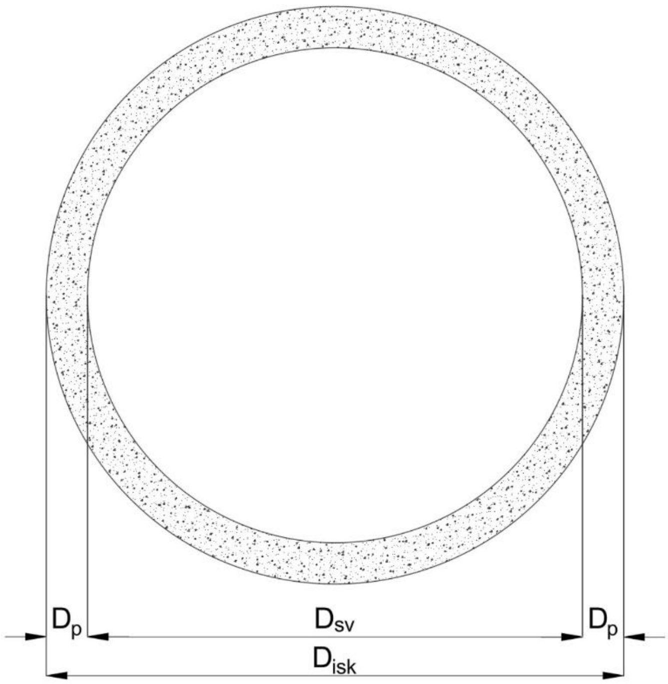

Figure 7 Diameters of different cross-sections

For the sake of unification, it is recommended that, based on the data obtained in this way, a standard cross-section be adopted according to the applicable standard (ISS, 1978), with dimensions no smaller than those calculated. The excavation diameter of the shaft will depend on the dimensions of the calculated and adopted support structure.

3 CONCLUSION

Modern mining science and practice face very complex problems related to the opening of new ore deposits at great depths. The opening of these deposits is typically done using shafts. It is always necessary to construct at least two shafts, one of which is the hoisting shaft.

In the field of hoisting shaft dimensioning, many decisions are still made based on experience. The choice of optimal shaft dimensions is crucial, with very little space for error, because in the case of a wrong decision, some of the most important conditions for exploitation will not be met. Often, the shaft is built with larger dimensions, with a certain safety factor. If the shaft dimensions are insufficient, it will conflict with the required mine capacity and will not meet the requirements in that regard. However, if the dimensions are too large, the costs and construction time exponentially increase. In addition to higher consumption of explosives, energy, and drilling elements, labor costs also increase because it takes longer to construct larger room profiles.

In this paper, the required volume of the skips was calculated, using the mine capacity and shaft depth as input data. Based on that, the required time for transporting the ore to the surface was calculated, and then the required skip volume to meet the capacity condition was determined. Unlike previous literature, where tabular values for distances and dimensions were adopted based on volume, this paper solves two equations with two unknowns to calculate the exact dimensions of the skips, from which the shaft diameter is graphically determined, and its cross-section is designed. This calculation can be used for multiple cases (two skips, skip with counterweight, etc.) and for different cross-sections (square, rectangular, or combined).

It should be noted that there are standards in this field that can help verify the calculated and graphically determined dimensions. There are also catalogs of designed, most used cross-sections.

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REFERENCES

ANTUNOVIĆ KOBLIŠKA, M. (1973) *Opšti rudarski radovi*, Građevinska knjiga, Belgrade.

HARTMAN HOWARD L., *SME Mining Engineering Handbook*, Society of Mining, Metallurgy and Exploration, Inc. Littleton, Colorado, 1992.

TORBICA, S. & LAPČEVIĆ V. (2020) *Metode podzemnog otkopavanja*, Faculty of Mining and Geology - University of Belgrade.

MUSIĆ, O, (2014) *Projektovanje podzemnih rudnika – skripta*, Rudarsko-geološko-građevinski fakultet - Univerzitet u Tuzli, Tuzla, Bosna i Hercegovina.

JOVANOVIĆ, P. (1982) *Izrada jamskih prostorija*, Univerzitet u Beogradu – Rudarsko-geološki fakultet, Beograd.

PAVLOVIĆ, V. (1963) *Transport i izvoz u rudnicima*, Zavod za izdavanje udžbenika, Beograd.

GRUJIĆ, M. (1999) *Transport i izvoz u rudnicima*, Univerzitet u Beogradu – Rudarsko-geološki fakultet, Beograd.

Službeni list SFRJ (1992) Pravilnik o tehničkim normativima pri prevozu ljudi i materijala oknima rudnika, Beograd.

ISS – Institute for Standardization of Serbia (1978) Standard SRPS B.Z0.108:1978, Belgrade