Review paper

CALCULATION METHOD AS THE SOURCE OF ERRORS IN CONNECTING AND ORIENTING UNDERGROUND MINING ROOMS

Aleksandar Ganić[1], Aleksandar Milutinović[1], Igor Miljanović[1], Zoran Gojković[1]

Abstract: The task of connecting survey and orienting mine rooms is to obtain the coordinates for the first point of the future underground traverse and the bearing of the first side of the underground traverse on the horizon that is being connected. The task is particularly complex when the connection survey is performed through a single vertical shaft. The accuracy of connection survey and orientation is influenced by the errors in given and measured values, but in addition a question may be asked if the applied calculation method influence the connection precision (accuracy) and if they do to what extent. If the connection through a single vertical shaft is completed by the connection rectangle method, the calculations are most often done by using the Hansen, Weiss or Hause method. This paper presents calculations of standard deviations of unknown values in a connecting rectangle based on the three methods, as well as standard deviations of the bearing of the first side of the underground traverse on an example.

1   INTRODUCTION

Connecting and orienting underground mining rooms through a single vertical shaft calls for projection of two shaft points from the terrain surface to the horizon being connected with the aid of two vertical plummets P_1 and P_2 (Figure 1). Plummets coordinates transferred from the terrain surface to the pit horizon can serve for positioning and orientation of the future underground traverse i.e. mine rooms. Plummets P_1 and P_2 with one and two unknown points A and B on the connecting horizon, respectively, are forming the connecting triangle, i.e. connecting rectangle with various shapes and relatively assorted sizes.

Straight lines observed in the connecting rectangle, are the foundation for the calculation of horizontal angles γ_1, γ_2, i.e. the angle γ, as well as angles δ_1, δ_2, i.e. the angle δ. These are the necessary measurements, which enables calculating the elements of the

---
[1] University of Belgrade – Faculty of Mining and Geology
Emails:aleksandar.ganic@rgf.bg.ac.rs;aleksandar.milutinovic@rgf.bg.ac.rs;
igor.miljanovic@rgf.bg.ac.rs; zoran.gojkovic@rgf.bg.ac.rs

connection triangle. In this case, the scale of the rectangle is defined by the known distance between the plumb lines.

In the connection rectangle, straight line of at least one side can be measured as well. The simples and safest solution is to measure the straight line d between the unknown points A and B. This measurement is redundant, enabling the control of the accuracy of measured values, or later the possibility for using the rigorous adjustment based on the least squares method.

The accuracy of connection and orientation of underground mines is influenced by the errors of measured values, as well as the errors of given values, or in this case, the error of projecting the plumb lines through a vertical shaft. However, the question may be asked if the applied mathematical method of rectangle calculation influence the accuracy of connection and orientation and if it does, to which extent.

In theory, the most often connection rectangles calculation methods applied are: Hansen (Woltermann, n.d.), Weiss (Schofield and Breach, 2007) and Hause (Davis et al. 1981). For the example of the same connection rectangle, all three calculation methods will be presented, meaning that the errors of unknown angles φ and ψ in the rectangle will be calculated, based on the various mathematical equations on which these three methods are relying upon. In this, we consider that the given values, namely the coordinates of plummets P_1 and P_2 are error free, i.e. that they have conditional error-free values.
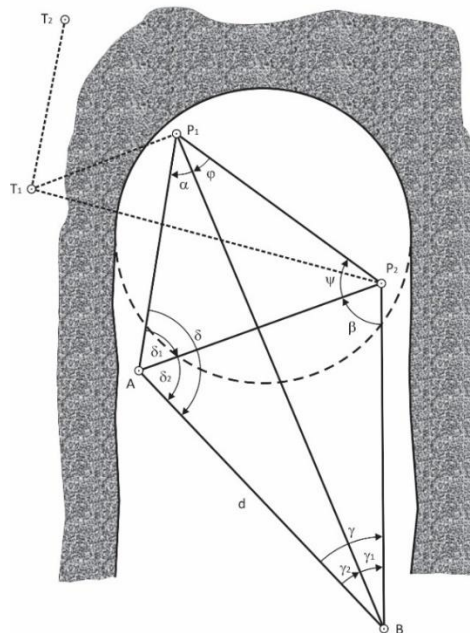

Figure 1 Connection rectangle

2   CONNECTION RECTANGLE CALCULATIONS

As it is widely known, in Geodesy and Mine Surveying straight lines are observed while horizontal angles are obtained from the differences of appropriate observed lines. Horizontal angles in the examples are:

$\gamma_1 = 13°21'16"$ $\delta_1 = 46°18'39"$

$\gamma_2 = 6°13'26"$ $\delta_2 = 119°17'12"$

$\gamma = 19°34'42"$ $\delta = 165°35'51"$

If the standard deviation of the observed straight lines amounts to $\sigma_P = \pm 2"$, then the standard deviations of angles γ_1; γ_2; γ; δ_1; δ_2; δ are mutually equal and amount to:

$$\sigma_{\gamma_1} = \sigma_{\gamma_2} = \sigma_\gamma = \sigma_{\delta_1} = \sigma_{\delta_2} = \sigma_\delta = \sigma_u = \pm 2\sqrt{2}" \tag{1}$$

The unknown angles α and β are obtained from the triangles:

from ΔP_1BA: $\alpha = 180° - \gamma_2 - \delta$

from ΔP_2BA: $\alpha = 180° - \gamma_2 - \delta$

Standard deviations of angles α and β, based on the error propagation law (Chandra, 2005) are:

$$\sigma_\alpha = \sqrt{(-1)^2\sigma_{\gamma_2}^2 + (-1)^2\sigma_\delta^2} = \sqrt{\sigma_{\gamma_2}^2 + \sigma_\delta^2} = \sqrt{2\sigma_u^2} = \sigma_u\sqrt{2} = \pm 4"$$

$$\tag{2}$$

$$\sigma_\beta = \sqrt{(-1)^2\sigma_{\gamma_2}^2 + (-1)^2\sigma_{\delta_2}^2} = \sqrt{\sigma_\gamma^2 + \sigma_{\delta_2}^2} = \sqrt{2\sigma_u^2} = \sigma_u\sqrt{2} = \pm 4"$$

2.1   Hansen method

Since $\varphi + \psi = \gamma_2 + \delta_2$ (from ΔP_1P_2B and ΔP_1P_2A), the semi-sum of the unknown angles φ and ψ is (Ganić et al., 2015):

$$\frac{\varphi + \psi}{2} = \frac{\gamma_2 + \delta_2}{2} \tag{3}$$

Standard deviation of the semi-sum amounts to:

$$\sigma_{\frac{\varphi+\psi}{2}} = \sqrt{k_{\gamma_2}^2 \cdot \sigma_{\gamma_2}^2 + k_{\delta_2}^2 \cdot \sigma_{\delta_2}^2} = \sqrt{\left(\frac{1}{2}\right)^2 \sigma_{\gamma_2}^2 + \left(\frac{1}{2}\right)^2 \sigma_{\delta_2}^2} = \frac{\sigma_u}{2}\sqrt{2} = \frac{2\sqrt{2}}{2}\sqrt{2} = \pm 2'' \qquad (4)$$

where:

$$k_{\gamma_2} = \frac{\partial\left(\frac{\varphi+\psi}{2}\right)}{\partial\gamma_2} = \frac{1}{2}; k_{\delta_2} = \frac{\partial\left(\frac{\varphi+\psi}{2}\right)}{\partial\delta_2} = \frac{1}{2}$$

According to the sine theorem, the following could be written:

$$\frac{sin\gamma_2 \cdot sin\delta_1 \cdot sin\varphi \cdot sin\beta}{sin\gamma_1 \cdot sin\delta_2 \cdot sin\alpha \cdot sin\psi} = 1 \qquad (5)$$

i.e.:

$$tan\mu = \frac{sin\varphi}{sin\psi} = \frac{sin\alpha \cdot sin\gamma_1 \cdot sin\delta_2}{sin\beta \cdot sin\gamma_2 \cdot sin\delta_1} = 1 \qquad (6)$$

The fictitious angle μ is:

$$\mu = arctan\left(\frac{sin\alpha \cdot sin\gamma_1 \cdot sin\delta_2}{sin\beta \cdot sin\gamma_2 \cdot sin\delta_1}\right) \qquad (7)$$

with the standard deviation:

$$\sigma_\mu = \sqrt{k_\alpha^2 \cdot \sigma_\alpha^2 + k_{\gamma_1}^2 \cdot \sigma_{\gamma_1}^2 + k_{\delta_2}^2 \cdot \sigma_{\delta_2}^2 + k_\beta^2 \cdot \sigma_\beta^2 + k_{\gamma_2}^2 \cdot \sigma_{\gamma_2}^2 + k_{\delta_1}^2 \cdot \sigma_{\delta_1}^2} = \pm 17.090'' \qquad (8)$$

where:

$$k_\alpha = \frac{\partial\mu}{\partial\alpha} = \frac{cos\alpha \cdot sin\gamma_1 \cdot sin\delta_2 \cdot sin\beta \cdot sin\gamma_2 \cdot sin\delta_1}{(sin\alpha \cdot sin\gamma_1 \cdot sin\delta_2)^2 + (sin\beta \cdot sin\gamma_2 \cdot sin\delta_1)^2} = +2.954$$

$$k_{\gamma_1} = \frac{\partial\mu}{\partial\gamma_1} = \frac{sin\alpha \cdot cos\gamma_1 \cdot sin\delta_2 \cdot sin\beta \cdot sin\gamma_2 \cdot sin\delta_1}{(sin\alpha \cdot sin\gamma_1 \cdot sin\delta_2)^2 + (sin\beta \cdot sin\gamma_2 \cdot sin\delta_1)^2} = +1.789$$

$$k_{\delta_2} = \frac{\partial\mu}{\partial\delta_2} = \frac{sin\alpha \cdot sin\gamma_1 \cdot cos\delta_2 \cdot sin\beta \cdot sin\gamma_2 \cdot sin\delta_1}{(sin\alpha \cdot sin\gamma_1 \cdot sin\delta_2)^2 + (sin\beta \cdot sin\gamma_2 \cdot sin\delta_1)^2} = -0.238$$

$$k_\beta = \frac{\partial \mu}{\partial \beta} = \frac{sin\alpha \cdot sin\gamma_1 \cdot sin\delta_2 \cdot cos\beta \cdot sin\gamma_2 \cdot sin\delta_1}{(sin\alpha \cdot sin\gamma_1 \cdot sin\delta_2)^2 + (sin\beta \cdot sin\gamma_2 \cdot sin\delta_1)^2} = -0.486$$

$$k_{\gamma_2} = \frac{\partial \mu}{\partial \gamma_2} = \frac{sin\alpha \cdot sin\gamma_1 \cdot sin\delta_2 \cdot sin\beta \cdot cos\gamma_2 \cdot sin\delta_1}{(sin\alpha \cdot sin\gamma_1 \cdot sin\delta_2)^2 + (sin\beta \cdot sin\gamma_2 \cdot sin\delta_1)^2} = -3.893$$

$$k_{\delta_1} = \frac{\partial \mu}{\partial \delta_1} = \frac{sin\alpha \cdot sin\gamma_1 \cdot sin\delta_2 \cdot sin\beta \cdot sin\gamma_2 \cdot cos\delta_1}{(sin\alpha \cdot sin\gamma_1 \cdot sin\delta_2)^2 + (sin\beta \cdot sin\gamma_2 \cdot sin\delta_1)^2} = -0.406$$

Since:

$$\frac{tan\frac{\varphi - \psi}{2}}{tan\frac{\varphi + \psi}{2}} = \frac{tan\mu - 1}{1 + tan\mu} = tan(\mu - 45°) \tag{9}$$

the semi-sum of the unknown angles is:

$$\frac{\varphi - \psi}{2} = arctan\left[tan\frac{\varphi + \psi}{2} \cdot tan(\mu - 45°)\right] \tag{10}$$

and their standard deviation:

$$\sigma_{\frac{\varphi - \psi}{2}} = \sqrt{k_{\frac{\varphi + \psi}{2}}^2 + \sigma_{\frac{\varphi + \psi}{2}}^2 + k_{\mu - 45°}^2 \cdot \sigma_\mu^2} = \pm 27.531" \tag{11}$$

where:

$$k_{\frac{\varphi + \psi}{2}} = \frac{\partial\left(\frac{\varphi - \psi}{2}\right)}{\partial\left(\frac{\varphi + \psi}{2}\right)} = \frac{tan(\mu - 45°)}{\left(cos\frac{\varphi + \psi}{2}\right)^2 + \left(sin\frac{\varphi + \psi}{2}\right)^2 \cdot tan^2(\mu - 45°)} = -1.042$$

$$k_{\mu - 45°} = \frac{\partial\left(\frac{\varphi - \psi}{2}\right)}{\partial(\mu - 45°)} = \frac{tan\left(\frac{\varphi + \psi}{2}\right)}{cos^2(\mu - 45°) + \left(tan\frac{\varphi + \psi}{2}\right)^2 \cdot sin^2(\mu - 45°)} = +1.606$$

The unknown angles φ and ψ are calculated based on their semi-sums and semi-differences, i.e.

$$\varphi = \frac{\varphi + \psi}{2} + \frac{\varphi - \psi}{2} \tag{12}$$

$$\psi = \frac{\varphi + \psi}{2} - \frac{\varphi - \psi}{2} \tag{13}$$

Their standard deviations are mutually equal and amount to:

$$\sigma_\varphi = \sigma_\psi = \sqrt{\sigma_{\frac{\varphi+\psi}{2}}^2 + \sigma_{\frac{\varphi-\psi}{2}}^2} = \pm 27.60''$$

(14)

2.2   Weiss method

Starting from the sine equation, the quotient M of the sines of unknown angles φ and ψ is calculated as:

$$M = \frac{\sin\psi}{\sin\varphi} = \frac{\sin\beta \cdot \sin\gamma_2 \cdot \sin\delta_1}{\sin\alpha \cdot \sin\gamma_1 \cdot \sin\delta_2}$$

(15)

Standard deviation of the quotient M is therefore:

$$\sigma_M = \sqrt{k_\beta^2 \cdot \sigma_\beta^2 + k_{\gamma_2}^2 \cdot \sigma_{\gamma_2}^2 + k_{\delta_1}^2 \cdot \sigma_{\delta_1}^2 + k_\alpha^2 \cdot \sigma_\alpha^2 + k_{\gamma_1}^2 \cdot \sigma_{\gamma_1}^2 + k_{\delta_2}^2 \cdot \sigma_{\delta_2}^2} = \pm 72.427''$$

(16)

where:

$$k_\beta = \frac{\partial M}{\partial \beta} = \cot\beta \frac{\sin\beta \cdot \sin\gamma_2 \cdot \sin\delta_1}{\sin\alpha \cdot \sin\gamma_1 \cdot \sin\delta_2} = +2.060$$

$$k_{\gamma_2} = \frac{\partial M}{\partial \gamma_2} = \cot\gamma_2 \frac{\sin\beta \cdot \sin\gamma_2 \cdot \sin\delta_1}{\sin\alpha \cdot \sin\gamma_1 \cdot \sin\delta_2} = +16.500$$

$$k_{\delta_1} = \frac{\partial M}{\partial \delta_1} = \cot\delta_1 \frac{\sin\beta \cdot \sin\gamma_2 \cdot \sin\delta_1}{\sin\alpha \cdot \sin\gamma_1 \cdot \sin\delta_2} = +1.719$$

$$k_\alpha = \frac{\partial M}{\partial \alpha} = -\cot\alpha \frac{\sin\beta \cdot \sin\gamma_2 \cdot \sin\delta_1}{\sin\alpha \cdot \sin\gamma_1 \cdot \sin\delta_2} = -12.521$$

$$k_{\gamma_1} = \frac{\partial M}{\partial \gamma_1} = \cot\gamma_1 \frac{\sin\beta \cdot \sin\gamma_2 \cdot \sin\delta_1}{\sin\alpha \cdot \sin\gamma_1 \cdot \sin\delta_2} = -7.580$$

$$k_{\delta_2} = \frac{\partial M}{\partial \delta_2} = \cot\delta_2 \frac{\sin\beta \cdot \sin\gamma_2 \cdot \sin\delta_1}{\sin\alpha \cdot \sin\gamma_1 \cdot \sin\delta_2} = +1.009$$

Since:

$$N = \varphi + \psi = \gamma_2 + \delta_2$$

(17)

and the standard deviation of the sum:

$$\sigma_N = \sigma_u \sqrt{2} = 2\sqrt{2} \cdot \sqrt{2} = \pm 4''$$

(18)

the unknown angle φ can be calculated from (15) and (17) following the equation:

$$\varphi = arctan\frac{sinN}{M + cosN} \tag{19}$$

Standard deviation of the φ angle is:

$$\sigma_\varphi = \sqrt{k_M^2 \cdot \sigma_M^2 + k_N^2 \cdot \sigma_N^2} = \pm 27.45" \tag{20}$$

where:

$$k_M = \frac{\partial\varphi}{\partial M} = -\frac{sinN}{1 + M^2 + 2M \cdot cosN} = +0.379$$

$$k_N = \frac{\partial\varphi}{\partial N} = \frac{1 + M \cdot cosN}{1 + M^2 + 2M \cdot cosN} = -0.021$$

From equations (15) and (17) angle ψ amounts to:

$$\psi = arctan\frac{M \cdot sinN}{1 + M \cdot cosN} \tag{21}$$

Standard deviation of the ψ angle is:

$$\sigma_\psi = \sqrt{k_M^2 \cdot \sigma_M^2 + k_N^2 \cdot \sigma_N^2} = \pm 27.75" \tag{22}$$

where:

$$k_M = \frac{\partial\psi}{\partial M} = -\frac{sinN}{1 + M^2 + 2M \cdot cosN} = +0.379$$

$$k_N = \frac{\partial\psi}{\partial N} = \frac{M(M + cosN)}{1 + M^2 + 2M \cdot cosN} = +1.021$$

2.3   Hause method

Hause method for solving the connection pit rectangle calls for measurement of the horizontal length between the unknown points *A* and *B*. For the purpose of comparison with the aforementioned methods, it will be considered that this length was measured error-free, i.e.

$$\overline{AB} = d = 4.50 \; m; \sigma_d = 0$$

Based on the length d, the coordinate of unknown points A and B were defined in the local coordinate system, with the adopted arbitrary bearing between them $v_{A'}^{B'} = 90°$, and with point coordinates:

$$y_{A'} = 500.00 \, m; x_{A'} = 500.00 \, m$$

$$y_{B'} = 504.50 \, m; x_{B'} = 500.00 \, m$$

which are also considered to be error- free.

By resecting, plummet P_1 coordinates are calculated in the local system as (Ogundare, 2016):

$$x'_{P_1} = \frac{(y_{A'} - y_{B'}) + x_{B'} \cdot tan\left(v_{B'}^{A'} + \gamma_2\right) - x_{A'} \cdot tan(v_{A'}^{B'} - \delta)}{tan(v_{B'}^{A'} + \gamma_2) - tan(v_{A'}^{B'} - \delta)} \tag{23}$$

$$y'_{P_1} = \frac{(x_{A'} - x_{B'}) + y_{B'} \cdot cot\left(v_{B'}^{A'} + \gamma_2\right) - y_{A'} \cdot cot(v_{A'}^{B'} - \delta)}{cot(v_{B'}^{A'} + \gamma_2) - cot(v_{A'}^{B'} - \delta)} \tag{24}$$

Standard deviation of the P_1 plummet coordinates is:

$$\sigma_{x_{P'_1}} = \sqrt{k_{\gamma_2}^2 \cdot \sigma_{\gamma_2}^2 + k_\delta^2 \cdot \sigma_\delta^2} = \pm 1.01 \, mm \tag{25}$$

where:

$$k_{\gamma_2} = \frac{\partial x_{P'_1}}{\partial \gamma_2} = \frac{sec^2(v_{B'}^{A'} + \gamma_2)[-(y_{A'} - y_{B'}) + (x_{A'} - x_{B'})tan(v_{A'}^{B'} - \delta)]}{[tan(v_{B'}^{A'} + \gamma_2) - tan(v_{A'}^{B'} - \delta)]^2} = -72.574 \, m$$

$$k_\delta = \frac{\partial x_{P'_1}}{\partial \delta} = \frac{sec^2(v_{A'}^{B'} - \delta)[-(y_{A'} - y_{B'}) + (x_{A'} - x_{B'})tan(v_{B'}^{A'} + \gamma_2)]}{[tan(v_{B'}^{A'} + \gamma_2) - tan(v_{A'}^{B'} - \delta)]^2} = -13.787 \, m$$

i.e.

$$\sigma_{y_{P'_1}} = \sqrt{k_{\gamma_2}^2 \cdot \sigma_{\gamma_2}^2 + k_\delta^2 \cdot \sigma_\delta^2} = \pm 0.12 \, mm \tag{26}$$

where:

$$k_{\gamma_2} = \frac{\partial y_{P'_1}}{\partial \gamma_2} = \frac{csc^2(v_{B'}^{A'} + \gamma_2)[(x_{A'} - x_{B'}) - (y_{A'} - y_{B'})cot(v_{A'}^{B'} - \delta)]}{[cot(v_{B'}^{A'} + \gamma_2) - cot(v_{A'}^{B'} - \delta)]^2} = -7.915 \, m$$

$$k_\delta = \frac{\partial y_{P'_1}}{\partial \delta} = \frac{csc^2(v_{A'}^{B'} - \delta)[(x_{A'} - x_{B'}) - (y_{A'} - y_{B'})cot(v_{B'}^{A'} + \gamma_2)]}{[cot(v_{B'}^{A'} + \gamma_2) - cot(v_{A'}^{B'} - \delta)]^2} = -3.541 \, m$$

Also, by resecting, the plummet P_2 coordinates are calculated in the local system:

$$x'_{P_2} = \frac{(y_{A'} - y_{B'}) + x_{B'} \cdot tan\left(v_{B'}^{A'} + \gamma\right) - x_{A'} \cdot tan(v_{A'}^{B'} - \delta_2)}{tan(v_{B'}^{A'} + \gamma) - tan(v_{A'}^{B'} - \delta_2)} \qquad (27)$$

$$y'_{P_2} = \frac{(x_{A'} - x_{B'}) + y_{B'} \cdot cot\left(v_{B'}^{A'} + \gamma\right) - y_{A'} \cdot cot(v_{A'}^{B'} - \delta_2)}{cot(v_{B'}^{A'} + \gamma) - cot(v_{A'}^{B'} - \delta_2)} \qquad (28)$$

Standard deviation of the plummet P_2 coordinates is:

$$\sigma_{x_{P'_2}} = \sqrt{k_\gamma^2 \cdot \sigma_{\gamma_2}^2 + k_{\delta_2}^2 \cdot \sigma_{\delta_2}^2} = \pm 0.25 \; mm \qquad (29)$$

where:

$$k_\gamma = \frac{\partial x_{P'_2}}{\partial \gamma} = \frac{sec^2(v_{B'}^{A'} + \gamma)[-(y_{A'} - y_{B'}) + (x_{A'} - x_{B'})tan(v_{A'}^{B'} - \delta_2)]}{[tan(v_{B'}^{A'} + \gamma) - tan(v_{A'}^{B'} - \delta_2)]^2} = -17.805 \; m$$

$$k_{\delta_2} = \frac{\partial x_{P'_2}}{\partial \delta_2} = \frac{sec^2(v_{A'}^{B'} - \delta_2)[-(y_{A'} - y_{B'}) + (x_{A'} - x_{B'})tan(v_{B'}^{A'} + \gamma)]}{[tan(v_{B'}^{A'} + \gamma) - tan(v_{A'}^{B'} - \delta_2)]^2} = -2.628 \; m$$

i.e.

$$\sigma_{y_{P'_2}} = \sqrt{k_\gamma^2 \cdot \sigma_\gamma^2 + k_{\delta_2}^2 \cdot \sigma_{\delta_2}^2} = \pm 0.11 \; mm \qquad (30)$$

where:

$$k_\gamma = \frac{\partial y_{P'_2}}{\partial \gamma} = \frac{csc^2(v_{B'}^{A'} + \gamma)[(x_{A'} - x_{B'}) - (y_{A'} - y_{B'})cot(v_{A'}^{B'} - \delta_2)]}{[cot(v_{B'}^{A'} + \gamma) - cot(v_{A'}^{B'} - \delta_2)]^2} = -6.332 \; m$$

$$k_{\delta_2} = \frac{\partial y_{P'_2}}{\partial \delta_2} = \frac{csc^2(v_{A'}^{B'} - \delta_2)[(x_{A'} - x_{B'}) - (y_{A'} - y_{B'})cot(v_{B'}^{A'} + \gamma)]}{[cot(v_{C'}^{D'} + \gamma) - cot(v_{D'}^{C'} - \delta_2)]^2} = -4.686 \; m$$

The unknown angle ψ is calculated from the difference of bearings between the sides $\overline{P_2P_1}$ and $\overline{P_2D}$ in the local coordinate system:

$$\psi = v_{P'_2}^{P'_1} - v_{P'_2}^{A'} = arctan\frac{y_{P'_1} - y_{P'_2}}{x_{P'_1} - x_{P'_2}} - arctan\frac{y_{A'} - y_{P'_2}}{x_{A'} - x_{P'_2}} \qquad (31)$$

Standard deviation of the angle ψ:

$$\sigma_\psi = \rho'' \sqrt{k_{y_{P'_1}}^2 \cdot \sigma_{y_{P'_1}}^2 + k_{x_{P'_1}}^2 \cdot \sigma_{x_{P'_1}}^2 + k_{y_{P'_2}}^2 \cdot \sigma_{y_{P'_2}}^2 + k_{x_{P'_2}}^2 \cdot \sigma_{x_{P'_2}}^2} = \pm 80.39'' \qquad (32)$$

where:

$$k_{y_{P'_1}} = \frac{\partial \psi}{\partial y_{P'_1}} = \frac{x_{P'_1} - x_{P'_2}}{\left(y_{P'_1} - y_{P'_2}\right)^2 + \left(x_{P'_1} - x_{P'_2}\right)^2} = -0.186 \, m^{-1}$$

$$k_{x_{P'_1}} = \frac{\partial \psi}{\partial x_{P'_1}} = \frac{y_{P'_1} - y_{P'_2}}{\left(y_{P'_1} - y_{P'_2}\right)^2 + \left(x_{P'_1} - x_{P'_2}\right)^2} = +0.357 \, m^{-1}$$

$$k_{y_{P'_2}} = \frac{\partial \psi}{\partial y_{P'_2}} = -\frac{x_{P'_1} - x_{P'_2}}{\left(y_{P'_1} - y_{P'_2}\right)^2 + \left(x_{P'_1} - x_{P'_2}\right)^2} + \frac{x_{A'} - x_{P'_2}}{\left(y_{A'} - y_{P'_2}\right)^2 + \left(x_{A'} - x_{P'_2}\right)^2} = -0.194 \, m^{-1}$$

$$k_{x_{P'_2}} = \frac{\partial \psi}{\partial x_{P'_2}} = -\frac{y_{P'_1} - y_{P'_2}}{\left(y_{P'_1} - y_{P'_2}\right)^2 + \left(x_{P'_1} - x_{P'_2}\right)^2} + \frac{y_{A'} - y_{P'_2}}{\left(y_{A'} - y_{P'_2}\right)^2 + \left(x_{A'} - x_{P'_2}\right)^2} = -0.571 \, m^{-1}$$

with angle φ:

$$\varphi = v_{P'_1}^{B'} - v_{P'_1}^{P'_2} = arctan \frac{y_{B'} - y_{P'_1}}{x_{B'} - x_{P'_1}} - arctan \frac{y_{P'_2} - y_{P'_1}}{x_{P'_2} - x_{P'_1}} \qquad (33)$$

Standard deviation ofthe angle φ:

$$\sigma_\sigma = \rho'' \sqrt{k_{y_{P'_1}}^2 \cdot \sigma_{y_{P'_1}}^2 + k_{x_{P'_1}}^2 \cdot \sigma_{x_{P'_1}}^2 + k_{y_{P'_2}}^2 \cdot \sigma_{y_{P'_2}}^2 + k_{x_{P'_2}}^2 \cdot \sigma_{x_{P'_2}}^2} = \pm 52.00'' \qquad (34)$$

where:

$$k_{y_{P'_1}} = \frac{\partial \varphi}{\partial y_{P'_1}} = -\frac{x_{B'} - x_{P'_1}}{\left(y_{B'} - y_{P'_1}\right)^2 + \left(x_{B'} - x_{P'_1}\right)^2} + \frac{x_{P'_2} - x_{P'_1}}{\left(y_{P'_2} - y_{P'_1}\right)^2 + \left(x_{P'_2} - x_{P'_1}\right)^2} = +0.200 \, m^{-1}$$

$$k_{x_{P'_1}} = \frac{\partial \varphi}{\partial x_{P'_1}} = \frac{y_{B'} - y_{P'_1}}{\left(y_{B'} - y_{P'_1}\right)^2 + \left(x_{B'} - x_{P'_1}\right)^2} - \frac{y_{P'_2} - y_{P'_1}}{\left(y_{P'_2} - y_{P'_1}\right)^2 + \left(x_{P'_2} - x_{P'_1}\right)^2} = -0.231 \, m^{-1}$$

$$k_{y_{P'_2}} = \frac{\partial \varphi}{\partial y_{P'_2}} = -\frac{x_{P'_2} - x_{P'_1}}{\left(y_{P'_2} - y_{P'_1}\right)^2 + \left(x_{P'_2} - x_{P'_1}\right)^2} = -0.186 \, m^{-1}$$

$$k_{x_{P'_2}} = \frac{\partial \varphi}{\partial x_{P'_2}} = \frac{y_{P'_2} - y_{P'_1}}{\left(y_{P'_2} - y_{P'_1}\right)^2 + \left(x_{P'_2} - x_{P'_1}\right)^2} = +0.357 \, m^{-1}$$

3   ANALYSIS OF THE OBTAINED RESULTS

Standard deviation values of unknown angles φ and ψ in the rectangle, calculated by employing the methods of Hansen, Weiss and Hause, are shown in Table 1. The table shows that the standard deviation of angles calculated by Hansen and Weiss methods are approximately equal, but significantly larger if calculated by Hause method. In the example shown, the standard deviation of angle φ calculated according to Hause method is almost two times higher in comparison to that of Hansen method, while the standard deviation of the angle ψ is larger almost three times.

Based on the calculated angles φ and ψ, i.e. their standard deviations, coordinates of points A and B, representing the first points of the future underground traverse were calculated, according to the resection method. Meaning that the bearing of the first side of the traverse v_A^B and its standard deviation $\sigma_{v_A^B}$ were calculated. As shown in table 1, standard deviation of the bearing of the first side is virtually equal, if the angles φ and ψ were calculated by using the Hansen or Weiss method, while the standard deviation of the bearing calculated by using the Hause method is almost 2.5 times higher.

Table 1 Standard deviation of angles φ, ψ and the bearing of the first side of the underground traverse

	σ_φ	σ_ψ	$\sigma_{v_A^B}$
Hansen	±27.60"	±27.60"	±98.01"
Weiss	±27.45"	±27.75"	±98.02"
Hause	±52.00"	±80.39"	±242.22"
Parametric adjustment	-	-	±92.77"

The fact that, due to using the Hause method, it was necessary to measure the horizontal length between the unknown points *A* and *B* in the pit, caused the total number of measured values to be 7 (6 straight lines and one horizontal length), i.e. to have one redundant measurement which can serve as the basis for the adjustment by least squares method (Wolf and Ghilani, 1997). By rigorous adjustment, the most probable values of the coordinates of unknown points *A* and *B* are calculated. Also, based on their standard deviations, the standard deviation of the bearing of the first side was calculated, amounting to value approximately 5% lower than the value obtained with Hansen i.e. Weiss method for calculating the angles φ and ψ.

4   CONCLUSION

One of the methods for connecting and orientation the underground mine rooms through a vertical shaft is the method of connection rectangle. Three most commonly applied procedures for calculating in the rectangle mentioned in the literature are the Hansen, Weiss and Hause method. All these methods are based upon the calculation of unknown angles φ and ψ in the rectangle by applying various mathematical equations. Standard deviations of unknown, calculated values are influenced by the applied mathematical method, aside from errors of given and measured values.

By disregarding the errors of given values, or treating them as conditionally true values, the standard deviations of unknown angles φ and ψ in the rectangle were calculated by applying the mentioned calculation methods based solely on the errors of measured values. The analysis has shown that standard deviation values of unknown angles of φ and ψ are minimal and approximately equal if the calculations are performed according to the method Hansen, i.e. Weiss. If calculated by the Hause method, standard deviation values of angles φ and ψ are almost two times larger for the angle φ, and even 3 times larger for the angle ψ in comparison to the methods of Hansen and Weiss.

Errors for this angles are causing the error of the bearing of first side of the underground traverse, i.e. the error of orientation for underground mine rooms. In the case of Hansen, i.e. Weiss method, the bearing error amounts to 98″, and if the Hause method is applied, the error amounts to 242″, meaning that it is almost 2.5 times higher.

This means that in connecting a single vertical shaft by the connection rectangle method, the Hause method should be avoided, because it causes significantly higher standard deviation of calculated values, and the calculation procedure is more complex in relation to the Hansen and the Weiss method.

On the other hand, since it is always practice to measure the straight line between the points A and B in the pit, as a control measure for the field measurements, this measurement is at the same time the redundant measurement in the rectangle, thus enabling the rigorous adjustment by applying the least square method. Rigorous adjustment provided the most probable values of unknown and measured values, hence the standard deviation of the bearing is minimal - 93″, i.e. by approximately 5% lower than in comparison to the Hansen and Weiss method.

The most accurate and precise connection and orientation of underground rooms by applying the connection rectangle method will be ensured if the redundant measurement is performed, as well as the rigorous adjustment by the least square method.

REFERENCES

CHANDRA, A.M. (2005) *Surveying: Problem Solving with Theory and Objective Type Questions*. New Delhi: New Age International (P) Ltd., Publishers.

DAVIS, R.E. et al. (1981) *Surveying: Theory and Practice*. 6th edition. New York: McGraw-Hill Publishing Company.

GANIĆ, A. et al. (2015) Standard deviation of the geodetic quadrilateral point coordinates determined by the application of Hansen's method. *Podzemni Radovi,* (26), pp. 55-64.

OGUNDARE, J.O. (2016) *Precision surveying: The principles and geomatics practice*. New Jersey: John Wiley & Sons.Inc.

SCHOFIELD, W. and BREACH, M. (2007) *Engineering Surveying*. 6th edition. UK: Elsevier.

WOLF, P.R. and GHILANI, C.D. (1997) *Adjustment Computations: Statistics and Least Squares in Surveying and GIS*. 3rd edition. New Jersey: John Wiley & Sons.Inc.

WOLTERMANN, M.L. (n.d.) *Two Survey Problems* [Online] Washington & Jefferson College. Available from: http://www2.washjeff.edu/users/mwoltermann/Dorrie/40.pdf [Accessed 03/27/2015].