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Original scientific paper

APLICATION FOR COORDINATE TRANSFOMATION BETWEEN GAUS - KRUGER PROJECTION - BESSEL ELLIPSOID AND UTM PROJECTION - WGS84 ELLIPSOID

APLIKACIJA ZA TRANSFORMACIJU KOORDINATA IZMEĐU GAUS - KRIGEROVE PROJEKCIJE - ELIPSOID BESELA I UTM PROJEKCIJE - WGS84 ELIPSOID

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Abstract: The physical surface of the earth has irregular shape which is not mathematically defined, therefore the shape of the Earth is approximated with mathematically defined surfaces such as ellipsoid and sphere. The developing of a global positioning systems, thus and modern navigation systems, as effect produce large amounts of data which contain the problem of homogeneity. This problem could be exceed if all the data are store in the same coordinate system. Hence the need for data transformation from local coordinate systems to the global coordinate systems. Global level implies WGS84 ellipsoid and UTM projection while national coordinate system of Republic Serbia is Gauss-Kruger with Bessel ellipsoid. This coordinate system of Republic Serbia on a global level has a local character. Applying appropriate mathematical models and functions it is possible to transform coordinates from one system to another and vice versa. The paper describes coordinate transformations from Gauss-Kruger coordinate system ellipsoid Bessel to UTM projection WGS84 ellipsoid and vice versa, and also an application which provides transformation of its kind that is made using open source environment. Name of the application is TRANS7_GK_UTM_GK and it can be found and used on the web page of the faculty for Mining and Geology under the link http://gk2utm.rgf.bg.ac.rs with a user guide.

Keywords: Gauss-Kruger, Besell, WGS84, UTM, coordinate transformation, r

Apstrakt: Fizička površ Zemlje je nepravilnog oblika koja matematički nije definisana zbog čega se oblik Zemlje aproksimira matematički definisanim površima kao što su elipsoid i sfera. Razvojem globalnih sistema za pozicioniranje, samim tim i sistema za navigaciju, za posledicu imamo ogromne količine podataka kod kojih se javlja i problem homogenosti u geoprostornom smislu. Ovaj problem može se prevazići ako bi se svi podaci nalazili u istom koordinatnom sistemu. Odatle i potreba da se podaci iz lokalnih (državnih) koordinatnih sistema transformišu u koordinatne sisteme koji su zastupljeni na globalnom (svetskom) nivou. U svetu je zastupljen elipsoid WGS84 i UTM projekcija, dok je državni koordinatni sistem Republike Srbije, koji posmatrano na globalnom nivou ima ulogu lokalnog karaktera, smešten na elipsoid Besela sa Gaus-Krigerovom

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projekcijom. Primenom odgovarajućih matematičkih modela i funkcija mogu se transformisati koordinate iz jednog sistema u drugi. U ovom radu prikazan je postupak transformacije iz jednog sistema u drugi, a i aplikacija koja omogućava transformaciju ovakve vrste koja je napravljena pomoću alata i softvera otvorenog koda pod nazivom TRANS7_GK-UTM-GK i dostupna na web adresi Rudarsko-geološkog fakulteta na linku http://gk2utm.rgf.bg.ac.rs sa uputstvom za upotrebu.

Ključne reči: Gaus-Kriger, Beselov elipsoid, WGS84, UTM, transformacija koordinata, r

1. INTRODUCTION

National networks and related data can be found in national projections which can be divided into zones, Also within a state, data can be located into different coordinate systems. EPSG (European Petroleum Survey Group – Oil & Gas Producers) has started collecting geodetic parameters since 1986th which are aviable on the web address http://www.epsg-registry.org/. This data set allows simple transformations between different coordinate systems, constantly evolving, progressing and updated.

Transformation leads to the data homogenization and it is therefore necessary to be done coordinate transformation between different systems. The paper describes coordinate transformations from Gauss-Kruger coordinate system ellipsoid Bessel to UTM projection WGS84 ellipsoid and vice versa, and also algorithm which can be used in „r“ open source software. Using the software package „shiny“ within „r“ software, TRANS7_GK-UTM-GK application is make to be used for users who don't need to know programming. Application TRANS7_GK_UTM_GK can be found and used on the web page of the faculty for Mining and Geology under the link http://gk2utm.rgf.bg.ac.rs with a user guide.

2. GAUSS-KRUGER PROJECTION AND UTM PROJECTION

Mathematically caused construction of the grid of meridians and parallels that serves as the mathematical basis for representation of the Earth's surface in the plane and for making maps is called a map projection. Position of the point is usually defined with geographical coordinates, latitude and longitude (φ,λ), and it's projection on the plane with cartesian coordinates (X,Y). Between (φ,λ) and (X,Y) exists a mathematical relation determined by choice of map projection, which corresponds to (Jovanović, 1983):

$$x=function1(lat,lon)=f_1(\varphi,\lambda) \quad y=function2(lat,lon)=g_1(\varphi,\lambda)$$

Table 1: Ellipsoid parameters (Wikipedia, 2012)

Elipsoid	Major radius (a)	Semi-minor axis (b)	Inverse flattening (1/f)
WGS 84	6 378 137.0m	≈ 6 356 752.314 245m	298.257 223 563
Bessel (1841)	6 377 397.155m	6 356 078.963m	299.1528153513233

2.1. Gauss-Kruger projection

Projection has called Gauss-Kruger because the theory concept had been developed by Gauss, and practical concept had done by Kruger on 1912th (Kruger 1912).

Gauss-Kruger projection is a traverse conformal projection on which the narrow meridian zones of the ellipsoid are mapped to the plane. In this projection central meridian is mapped as a straight line (X – axis of the rectangular coordinate system) while the scale on the central meridian should remain constant and equal to one.

National coordinate system implies Gauss-Kruger projection with Bessel ellipsoid (Table 1) and Hermannskogel datum. The largest part of the surface of the Republic of Serbia is mapped to a projection plane for whose x-axis has been taken projection of the 21° (longitude) meridian east from Greenwich and the width of the zone 3°. Y-axis represent equator projected on the plane.

2.2. UTM projection

The Universal Transverse Mercator (UTM) is modified Gauss-Kruger projection. With this projection it has been achieved conditions that all Earth surface can be represented with one coordinate system with limitation within polar areas where has adopted Universal Polar Stereographic projection (UPS). The width of the zones is 6o and whole world has divided into sixty zones.

All formulas for mathematical explanation into Gauss-Kruger projection are valid and for UTM projection with change into scale along the central meridian which is equal to 0.9996 and y-axis starts with 500000 instead of 7500000.

3. COORDINATE TRANSFORMATION

3.1. Transformation from Gauss-Kruger into UTM projection

Parameters (Radojčić, 2008):

1. Linear eccentricity

   $E = \sqrt{a^2 - b^2}$,

2. First eccentricity:

   $e = \frac{E}{a}$,

3. Second eccentricity

   $e' = \frac{E}{b}$,

4. Flattening

   $f = \frac{a - b}{a}$,

5. Radius of the curvature – poles

   $c = \frac{a^2}{b}$,

6. Radius of the curvature – meridian

   $M = \frac{a\left(1 - e^2\right)}{\sqrt{\left(1 - e^2 \sin^2 B\right)^3}}$,

7. Radius of the curvature – vertical

   $N = \frac{a}{\sqrt{1 - e^2 \sin^2 B}}$,

8. Radius of the curvature – parallel

   $R = N \cos B$,

   B – geodetic latitude,

   L – geodetic longitude,

9. Third ellipsoidal flattening

   $n = \frac{a - b}{a + b}$,

   λ_o – geodetic length of the prime meridian ($\lambda_o = 21^o$),

10. Radius of the curvature – prime vertical:

    $N_o = \frac{a}{\sqrt{1 - e^2 \sin^2 B_o}}$,

Auxiliary values:

$$t_o = \tan B_o , \quad \eta_o^2 = e'^2 \cos^2 B_o$$

Step one: Unmodulation of the coordinates (Gauss-Kruger projection) ***((У,X)***$_{GK}$ ***–*** $(\bar{У},\bar{X})_{GK}$***),***

$$\bar{Y} = \frac{Y-K}{0{,}9999}; \qquad \bar{X} = \frac{X}{0{,}9999}; \qquad (K = 7\ 500\ 000, m_0 = 0.9999) \qquad (1)$$

Step two: Transformation of rectangular unmodulated coordinates (Gauss-Kruger projection) into ellipsoidal-geodetic coordinates (Besell ellipsoid) ((y,x)GK - (B, L)Bes).

$$\begin{aligned} B = B_o &+ \frac{t_o}{2N_o^2}(-1-\eta_o^2)y^2 + \\ &+ \frac{t_o}{24N_o^4}(5+3t_o^2+6\eta_o^2-6t_o^2\eta_o^2-3\eta_o^4-9t_o^2\eta_o^4)y^4 + \\ &+ \frac{t_o}{720N_o^6}(-61-90t_o^2-45t_o^4-107\eta_o^2+162t_o^2\eta_o^2+45t_o^4\eta_o^2)y^6 + \\ &+ \frac{t_o}{40320N_o^8}(1385+3633t_o^2+4095t_o^4+1575t_o^6)y^8 + \dots \end{aligned} \qquad (2)$$

$$\begin{aligned} L = L_o &+ \frac{1}{N_o\cos B_o}y + \frac{1}{6N_o^3\cos B_o}(-1-2t_o^2-\eta_o^2)y^3 + \\ &+ \frac{1}{120N_o^5\cos B_o}(5+28t_o^2+24t_o^4+6\eta_o^2+8t_o^2\eta_o^2)y^5 + \\ &+ \frac{1}{5040N_o^7\cos B_o}(-61-662t_o^2-1320t_o^4-720t_o^6)y^7 + \dots \end{aligned} \qquad (3)$$

Where:

$$B_o = \frac{x}{\bar{\alpha}} + \bar{\beta}\sin\frac{2x}{\bar{\alpha}} + \bar{\gamma}\sin\frac{4x}{\bar{\alpha}} + \bar{\delta}\sin\frac{6x}{\bar{\alpha}} + \bar{\varepsilon}\sin\frac{8x}{\bar{\alpha}} + \dots \qquad (4)$$

$$\bar{\alpha} = \frac{a+b}{2}(1+\frac{1}{4}n^2+\frac{1}{64}n^4+\dots) \qquad (5)$$

$$\bar{\beta} = \frac{3}{2}n - \frac{27}{32}n^3 + \frac{69}{512}n^5 + \dots \qquad (6)$$

$$\bar{\gamma} = \frac{21}{16}n^2 - \frac{55}{32}n^4 + \dots \qquad (7)$$

$$\bar{\delta} = \frac{151}{96}n^3 - \frac{417}{128}n^5 + ... \quad (8)$$

$$\bar{\varepsilon} = \frac{1097}{512}n^4 + ... \quad (9)$$

Step three: Transformation of ellipsoidal-geodetic coordinates (Besell ellipsoid) into rectangular coordinates on the ellipsoid of Bessel ***((B, L)$_{Bes}$ - (X, Y, Z)$_{Bes}$),***

$$x = (N + h)\cos B \cos L \quad (10)$$

$$y = (N + h)\cos B \sin L \quad (11)$$

$$z = \left(N\left(1 - e^2\right) + h\right)\sin B \quad (12)$$

Where:

$$N = \frac{a^2}{\sqrt{a^2 \cos^2 B + b^2 \sin^2 B}}$$

Step four: Transformation of rectangular coordinates on the ellipsoid of Bessel into rectangular coordinates on the WGS84 ellipsoid ((X, Y, Z) Bes - (X, Y, Z) WGS84), (Ganić et al., 2014).

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{WGS} = (1+k)\begin{bmatrix} 1 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 1 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{Bessel} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \quad (13)$$

Table 2: Transformation parameters (Republic Geodetic Authority, 2017)

	Transformation parameters	Standard deviation
Translation x	t_x = 574.040907 m	$\sigma(t_x)$ = 0.015 m
Translation y	t_y = 170.129711 m	$\sigma(t_y)$ = 0.015 m
Translation z	t_z = 401.553949 m	$\sigma(t_z)$ = 0.015 m
Rotation x	ε_x = -4.88790271"	$\sigma(\varepsilon_x)$ = 0.032"
Rotation y	ε_y = 0.66492609"	$\sigma(\varepsilon_y)$ = 0.049"
Rotation z	ε_z = 13.24674576"	$\sigma(\varepsilon_z)$ = 0.044"
Scale	κ = 6.88937746 ppm	$\sigma(\kappa)$ = 0.106 ppm

Step five: Transformation of rectangular coordinates on the WGS84 ellipsoid into the geographical coordinates on the WGS84 ellipsoid ((X, Y, Z)WGS84 -(B, L)WGS84),

$$\lambda = \arctan\frac{y}{x} \qquad [rad] \tag{14}$$

$$B_o = \arctan\frac{z}{D\left(1-e^2\right)} \tag{15}$$

$$N_i = \frac{a}{\sqrt{1-e^2\sin^2 B_i}} \qquad i = 0,1,2... \tag{16}$$

$$h_i = \frac{D}{\cos B_i} - N_i \qquad i = 0,1,2... \tag{17}$$

$$\varphi_i = \arctan\frac{z}{D}\left(1-e^2\frac{N_i}{N_i+h_i}\right)^{-1} \qquad i = 0,1,2... \tag{18}$$

Radius of the parallel:

$$D = \sqrt{x^2+y^2} \tag{19}$$

It should be noted that this is an iterative process where iteration process ends when the geodetic latitude from the two successive iterations have the same values.

Step six: Transformation of the geographical coordinates on the WGS84 ellipsoid into rectangular coordinates UTM projection.

Length of the arc of the meridian

$$G(B) = \alpha\left(B+\beta\sin 2B+\gamma\sin 4B+\delta\sin 6B+\varepsilon\sin 8B+...\right) \tag{20}$$

Coefficients:

$$\alpha = \frac{a+b}{2}\left(1+\frac{1}{4}n^2+\frac{1}{64}n^4+...\right), \tag{21}$$

$$\beta = -\frac{3}{2}n+\frac{9}{16}n^3-\frac{3}{32}n^5+..., \tag{22}$$

$$\gamma = \frac{15}{16}n^2 - \frac{15}{32}n^4 + ... \tag{23}$$

$$\delta = -\frac{35}{48}n^3 + \frac{105}{256}n^5 - ..., \tag{24}$$

$$\varepsilon = \frac{315}{512}n^4 + ..., \tag{25}$$

$$n = \frac{a-b}{a+b}, \tag{26}$$

$$\begin{aligned} \bar{x} = \bar{X} + \frac{t}{2}N_i \cos^2\varphi_{Bes} \cdot l^2 + \frac{t}{24}N_i \cos^4\varphi_{Bes}\left(5 - t^2 + 9\eta^2 + \eta^4\right)l^4 + \\ + \frac{t}{270}N_i \cos^6\varphi_{Bes}\left(61 - 58t^2 + t^4 + 270\eta^2 - 330t^2\eta^2\right)l^6 + \\ + \frac{t}{40320}N_i \cos^8\varphi_{Bes}\left(1385 - 3111t^2 + 543t^4 - t^6\right)l^8 + ... \end{aligned} \tag{27}$$

Where:

$$\bar{X} = G(B) \tag{28}$$

$$\begin{aligned} \bar{y} = N_i \cos\varphi_{Bes} \cdot l + \frac{1}{6}N_i \cos^3\varphi_{Bes}\left(1 - t^2 + \eta^2\right)l^3 + \\ + \frac{1}{120}N_i \cos^5\varphi_{Bes}\left(5 - 18t^2 + t^4 + 14\eta^2 - 58t^2\eta^2\right)l^5 + \\ + \frac{1}{5040}N_i \cos^7\varphi_{Bes}\left(61 - 479t^2 + 179t^4 - t^6\right)l^7 + ... \end{aligned} \tag{29}$$

$$t = \tan\varphi,\ \eta = e'^2 \cos^2\varphi \tag{30}$$

$$l = \lambda - \lambda_0 \tag{31}$$

Step seven: Transformation of unmodulated coordinates into modulated coordinates – UTM projection ***(($\bar{У}$,$\bar{X}$)$_{УTM}$ – (У,X)$_{УTM}$),***

$$x = N = \bar{x} \cdot m \tag{32}$$

$$y = E = \bar{y} \cdot m + 500000 \tag{33}$$

$$m = 0{,}9996 \quad (X= N,\ У=E) \tag{34}$$

3.2 Transformation from UTM projection into Gauss-Kruger projection

Transformation from UTM projection into Gauss-Kruger projection is reverse process. It should be noted that now process starts in UTM projection and parameters for UTM projection should be used, i.e. (Lazić, 2010):

$$(E, N)_{UTM} - (B, L)_{WGS84} - (X, Y, Z)_{WGS84} - (X, Y, Z)_{Besel} - (B, L)_{Besel} - (y, x)_{GK}$$

Step one: Unmodulation of the coordinates (UTM projection),

$$y' = \frac{(E - 500000)}{m}; \quad x' = \frac{N}{m} \tag{35}$$

Step two: Transformation from y', x' into $(B, L)_{WGS84}$

For all calculations should be used parameters of the WGS84 ellipsoid (Table 1) and formulas from the second step above.

Step three: Transformation from $(B, L)_{WGS84}$ into $(X, Y, Z)_{WGS84}$

Should be used all equations from third step above.

Step four: Transformation from $(X, Y, Z)_{WGS84}$ - $(X, Y, Z)_{Besel}$

Should be used all equations from previous step four, only exception is that transformations parameters in reversed process (minus sign) should be used (**Table 2**)

Step five: Transformation from $(X, Y, Z)_{Besel}$ into $(B, L)_{Besel}$

Same equations from the step five above with parameters of the Bessel ellipsoid.

Step six: Transformation from $(B, L)_{Besel}$ into $(\overline{Y}, \overline{X})_{GK}$

Should be used all equations from the previous step six above.

Step seven: Transformation from $(\overline{Y}, \overline{X})_{GK}$ into modulated $(Y, X)_{GK}$,

$$Y = \overline{Y} \cdot m_0 + 7500000; \quad X = \overline{X} \cdot m_0; \quad m_0 = 0.9999 \tag{36}$$

4. APLICATION FOR COORDINATE TRANSFORMATION USING „Shiny“ PACKAGE AS PART OF THE „r“ OPEN SOURCE SOFTWARE

„r“ is an open source software and programming language available at web adress https://cran.r-project.org/, where it can be downloaded and used freely. „r“ is an integrated suite of software facilities for data manipulation, calculations and graphical display. Also „r“ is programming language that provides interface to other programming languages, as well as ready-made algorithms in many areas. It can be said that „r“ is an evolution of „s" language created by Bell laboratory. At the moment „r" have a 29 standard packages and 9987 packages available in library which can be added (Cran.r-project.org, 2017).

„Shiny" is a package witin the „r" open source software that makes easy to build interactive web applications. Shiny features works in the r environment and the main goal is that users do not need to know „r" (Shiny.rstudio.com, 2017).

Architecture of the „Shiny" application consists two components:

1. User interface (ui.r) component - is „r" script file which control the appearance and content of the application. It must be defined in the source script file with name ui.r.
2. Server component (server.r) is an engine of the application, and contains all the necessary instructions for the proper functioning of the application. Also should be defined in the source script file under the name server.r.

Besides „Shiny" package also have been used „rgdal" and „sp" package. „rgdal" package allows importing of vector and raster data into „r", which can be easily converted to spatial data (Cran.r-project.org, 2016).

Many packages within the „r" contained own presentations of the spatial data, what caused problems in the data exchange between other „r" packages and external applications. With this package „r" is more coherent for analyzing different types of spatial data, and data conversion to external applications is very simple (Cran.r-project.org, 2016).

Ui.r and server.r script file can be found on the web page of the Underground Mining Engineering Journal.

Staring application

Every „Shiny" application have same structure: two r script files saved into same folder with names ui.r and sarver.r. Aplication can be started through the button „runApp" that will appear when in the Rstudio are open these two files (Figure 1). Starting the application on the „runApp" button, application will be shown into Rstudio or web browser (Figure 2).

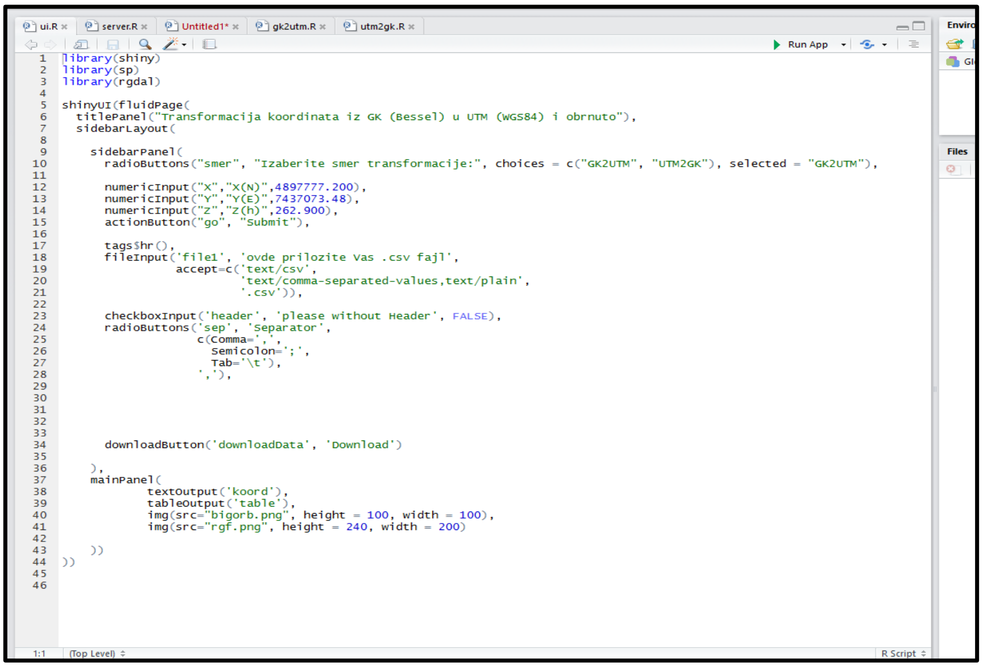


Figure 1 Starting the application

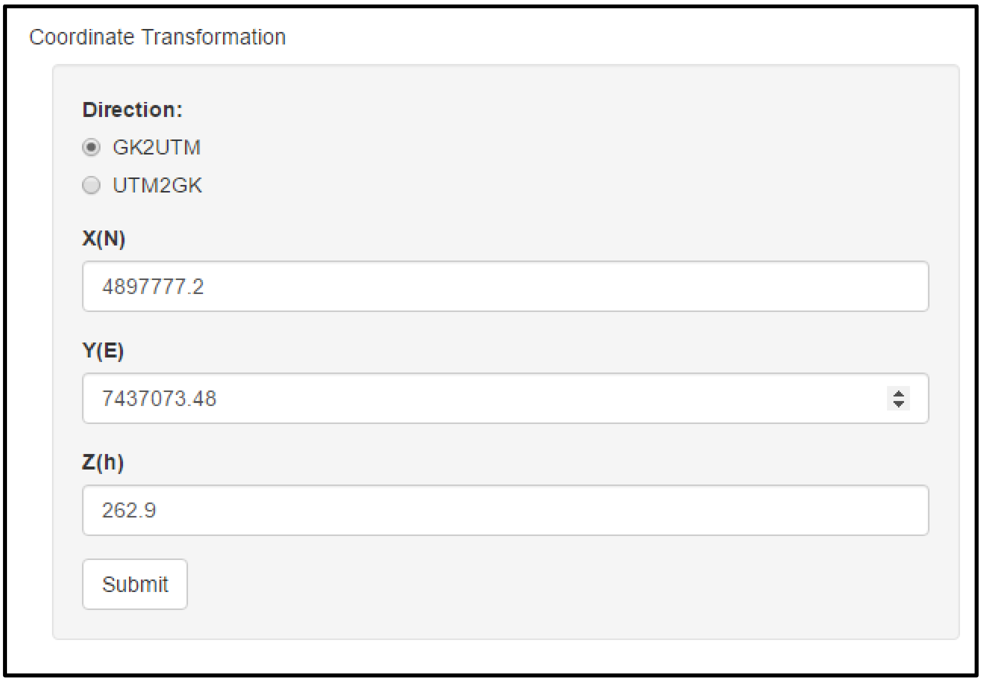


Figure 2 Application in window

4.1 Application TRANS7_GK-UTM-GK

ui.r component

shinyUI(fluidPage( )) – represent code which is minimum to be created „Shiny“ application. Result is an empty application with an empty user interface.

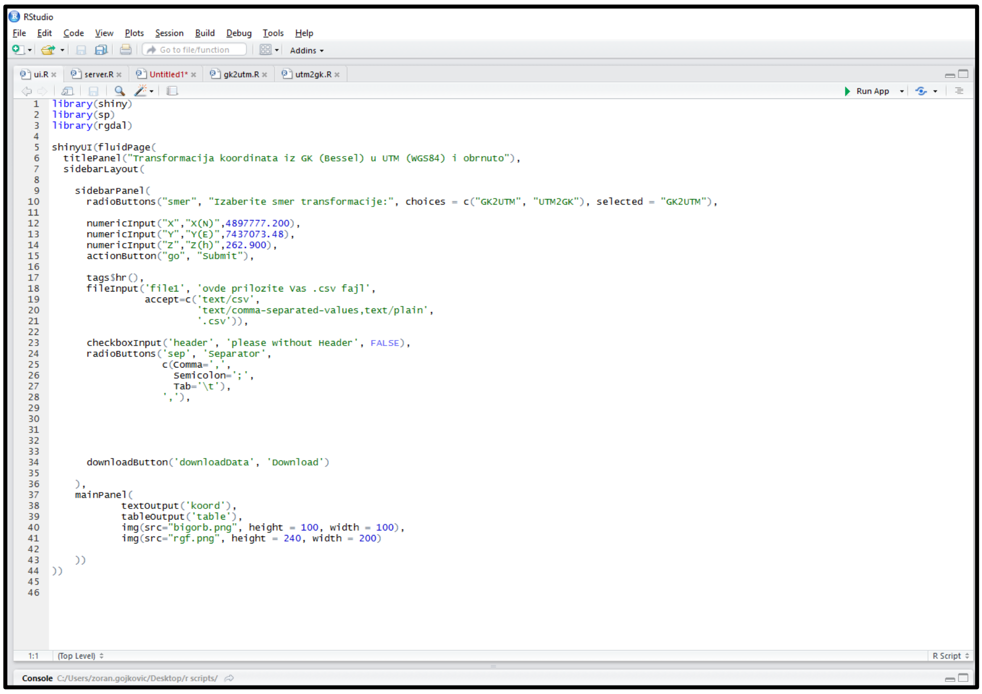


Figure 3 ui.r component

„Shiny“ ui.r script file use function *fluidpage* to create display which can be adjusted automatically. Figure 3 shows script file for making user interface. Besides those contents it is also important to provide location for output data.

„Shiny“ contains prebuilt widgets where every widget represent „r“ function. Most used widgets are counters, buttons etc which can be easily added into main or another panel. Every widget requires few arguments where main arguments are name and label.

server.r component

Placing widgets into ui.r script file provides interactive content into user interface where users can process their data. Next step is to build output data which can be downloaded by the user. This can be done with r code saved into the script file server.r. „Shiny“

contains lot of different widgets which transform r objects into the output data and sending them to the user (Figure 4).

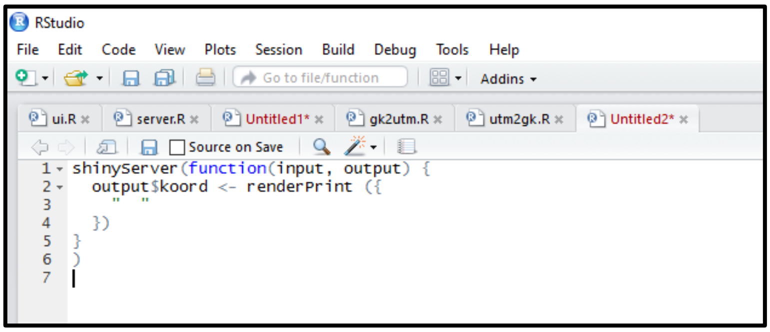


Figure 4 Server.r script file where can be placed r code

4.2. User manual for application TRANS7_GK-UTM-GK

Generally, application TRANS7_GK-UTM-GK consists of two parts. Figure 5 represent the first part of application and it can be used for individual point transformation. User must choose direction of transformation. Direction GK2UTM means that user should write Gauss-Kruger coordinates into fields X(N), Y(E) и Z(h) (zone 7). By clicking *Submit* button, on the screen will be shown coordinates transformed into UTM projection (Figure 6). Changing the direction into UTM2GK user must write UTM coordinates into corresponding fields and result will be Gauss-Kruger coordinates.

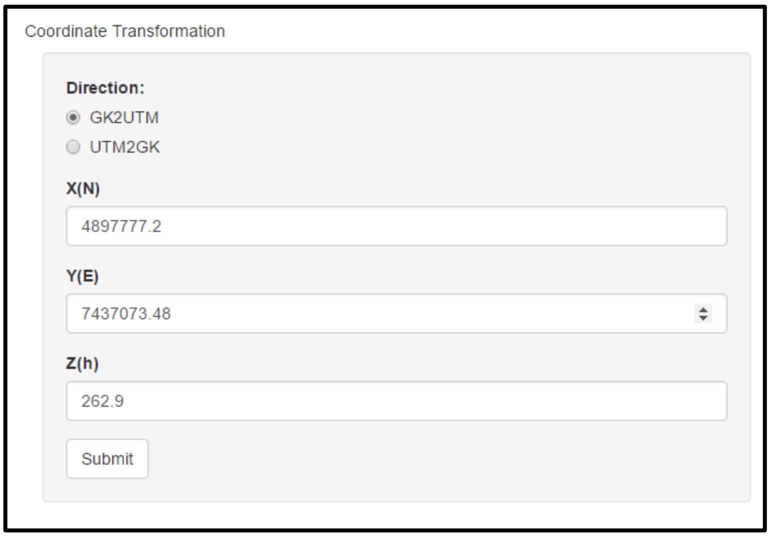


Figure 5 First part of the application

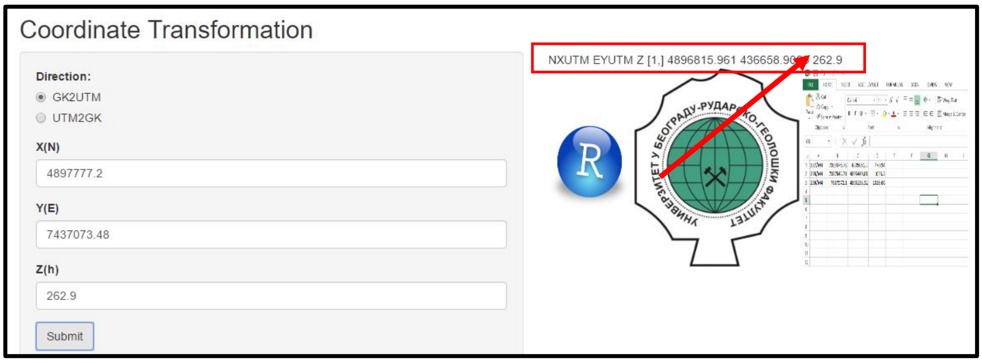
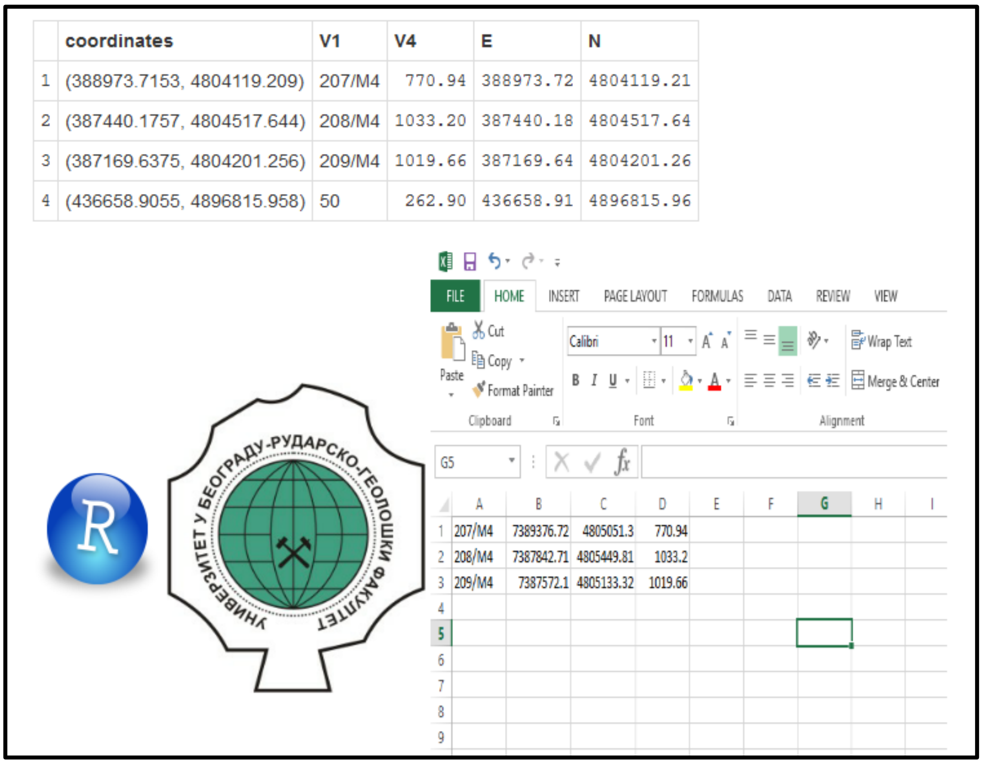


Figure 6 Coordinates after transformation

In the second part of the application (Figure 7) user attach file with coordinates, application then transform coordinates in selected direction and after that user can download file with transformed coordinates, in csv format, clicking on the button *Download*. In the right part of the application it will be shown table with transformed coordinates after attaching file for user preview (Figure 8).

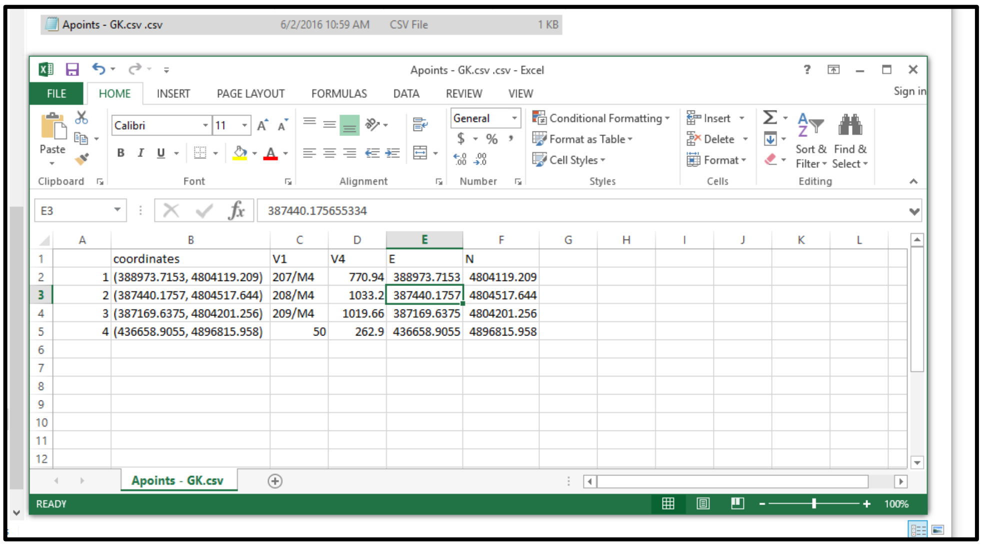
Figure 7 Second part of the application



	coordinates	V1	V4	E	N
1	(388973.7153, 4804119.209)	207/M4	770.94	388973.72	4804119.21
2	(387440.1757, 4804517.644)	208/M4	1033.20	387440.18	4804517.64
3	(387169.6375, 4804201.256)	209/M4	1019.66	387169.64	4804201.26
4	(436658.9055, 4896815.958)	50	262.90	436658.91	4896815.96

	A	B	C	D
1	207/M4	7389376.72	4805051.3	770.94
2	208/M4	7387842.71	4805449.81	1033.2
3	209/M4	7387572.1	4805133.32	1019.66

Figure 8 Table with transformed coordinates



	A	B	C	D	E	F
1		coordinates	V1	V4	E	N
2	1	(388973.7153, 4804119.209)	207/M4	770.94	388973.7153	4804119.209
3	2	(387440.1757, 4804517.644)	208/M4	1033.2	387440.1757	4804517.644
4	3	(387169.6375, 4804201.256)	209/M4	1019.66	387169.6375	4804201.256
5	4	(436658.9055, 4896815.958)	50	262.9	436658.9055	4896815.958

Figure 9 Downloaded file opened into Microsoft excel

Figure 9 shows output file into csv format downloaded by user and opened into Microsoft Excel. Columns represents respectively:

1. V1 –point name,
2. V4 – height,
3. E – easting (Y),
4. N – northing (X).

It should be noted that the starting screen contain picture how it should be prepared input csv file.

5. CONCLUSION

This kind of transformations includes three translations, three rotations and changing scale. GIS software's very often use simplified transformation models which are in some cases good enough for certain purposes. Selection of the coordinate system is one of the first steps in establishing GIS and it should be done with particular attention, because well planned and adequately adopted projection is a base for establishing GIS. This paper shows all equations for coordinate transformation from Gauss-Kruger into UTM and vice versa. Due to the large number of the equations and the length of the whole process best way for visualizing and use was to make single interactive application. Such application has made in the „r“ software using package „Shiny“ and it can be found on the web site of the Faculty for Mining and Geology under the url gk2utm.rgf.bg.ac.rs.

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